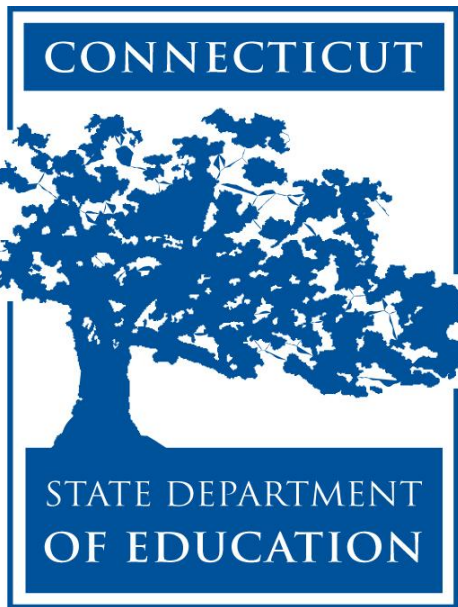


# Claim 1: Mathematics Concepts and Procedures

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# Claim 1: Concepts and Procedures

“Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.”

# Smarter Balanced Cognitive Rigor Matrix

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
<b>Remember</b>	- Recall conversions, terms, facts			
<b>Understand</b>	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
<b>Apply</b>	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question - Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
<b>Analyze</b>	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets
<b>Evaluate</b>			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
<b>Create</b>	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation

# Rationale for Claim 1

- This claim addresses procedural skills and the conceptual understanding on which developing skills depend. It is important to assess how aware students are of how concepts link together and why mathematical procedures work the way they do.
- Central to understanding this claim is making the connection to these elements of the mathematical practices as stated in the CCSSM.
  - **MP5. Use appropriate tools strategically.**
  - **MP6. Attend to precision.**
  - **MP7. Look for and make use of structure.**
  - **MP8. Look for and express regularity in repeated reasoning.**

# MP5: Use appropriate tools strategically

Mathematically proficient students:

- Detect possible errors by strategically using estimation and other mathematical knowledge.
- Use technological tools to explore and deepen their understanding of concepts.

## MP6: Attend to precision

Mathematically proficient students:

- State the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- Specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- Calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context.
  - Older students should be able to examine claims and make explicit use of definitions.

## MP7: Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
- Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
- Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for the distributive property.
- In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.



## MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated.
- look for both general methods and shortcuts.
- Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeated decimal.
- Middle school students might abstract the equation  $(y-2)/(x-1)=3$  by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1, 2) with a slope 3.
- maintain oversight of the process of solving a problem, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

# Smarter Balanced Assessment Targets

- Provide more detail about the range of content and Depth of Knowledge levels.
- Intended to support the development of high-quality items and tasks that contribute evidence to the claims.
- The cluster level headings of the standards in the CCSSM are the Assessment Targets for Claim 1

# Claim 1 Assessment Targets

## Grade 7 SUMMATIVE ASSESSMENT TARGETS

### Providing Evidence Supporting Claim #1

**Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.**

Content for this claim may be drawn from any of the Grade 7 clusters represented below, with a much greater proportion drawn from clusters designated “m” (major) and the remainder drawn from clusters designated “a/s” (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4.<sup>18</sup> Grade level content emphases are summarized in Appendix A and CAT sampling proportions for Claim 1 are given in Appendix B.

#### Ratios and Proportional Relationships (7.RP)

**Target A [m]: Analyze proportional relationships and use them to solve real-world and mathematical problems. (DOK 1, 2)**

Tasks for this target will require students to identify and represent proportional relationships in various formats (tables, graphs, equations, diagrams, verbal descriptions) and interpret specific values in context. (See 7.G Target E for possible context.) Other tasks will require students to compute unit rates, including those associated with ratios of fractions.

Multistep problems involving ratio and percent will be assessed by tasks in Claims 2 and 4.

# Grade 3 Targets for Claim 1

- A. Represent and solve problems involving multiplication and division
- B. Understand properties of multiplication and the relationship between multiplication and division
- C. Multiply and divide within 100
- D. Solve problems involving the four operations, and identify and explain patterns in arithmetic
- E. Use place value understanding and properties of operations to perform multi-digit arithmetic
- F. Develop understanding of fractions as numbers
- G. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects
- H. Represent and interpret data
- I. Geometric measurement: understand concepts of area and relate area to multiplication and to addition
- J. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures
- K. Reason with shapes and their attributes

# Smarter Balanced Assessment Targets

COMMON CORE STATE STANDARDS

## Operations and Algebraic Thinking

3.OA

### Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .*
2. Interpret whole-number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.<sup>1</sup>
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations  $8 \times ? = 48$ ,  $5 = \square \div 3$ ,  $6 \times 6 = ?$ .*

# Grade 6 Targets for Claim 1

- A. Understand ratio concepts and use ratio reasoning to solve problems
- B. Apply and extend previous understandings of multiplication and division to divide fractions by fractions
- C. Apply and extend previous understandings of numbers to the system of rational numbers
- D. Compute fluently with multi-digit numbers and find common factors and multiples
- E. Apply and extend previous understandings of arithmetic to algebraic expressions
- F. Reason about and solve one-variable equations and inequalities
- G. Represent and analyze quantitative relationships between dependent and independent variables
- H. Solve real-world and mathematical problems involving area, surface area, and volume
- I. Develop understanding of statistical variability
- J. Summarize and describe distributions

# High School Targets for Claim 1

- A. Extend the properties of exponents to rational exponents
- B. Use properties of rational and irrational numbers
- C. Reason quantitatively and use units to solve problems
- D. Interpret the structure of expressions
- E. Write expressions in equivalent forms to solve problems
- F. Perform arithmetic operations on polynomials
- G. Create equations that describe numbers or relationships
- H. Understand solving equations as a process of reasoning and explain the reasoning
- I. Solve equations and inequalities in one variable
- J. Represent and solve equations and inequalities graphically
- K. Understand the concept of a function and use function notation
- L. Interpret functions that arise in applications in terms of a context
- M. Analyze functions using different representations
- N. Build a function that models a relationship between two quantities
- O. Define trigonometric ratios and solve problems involving right triangles
- P. Summarize, represent and interpret data on a single count or measurement variable

# Essential Properties of Claim 1 Items

- Claim 1 items will be assessed on the computer-adaptive (CAT) portion of the summative assessment, using a combination of :
  - multiple choice, single correct response; multiple choice, multiple correct response; matching tables; equation/numeric; graphing; drag and drop; and fill-in table items that focus on explaining and applying mathematical concepts and carrying out mathematical procedures with precision and fluency.



# The Mathematics Assessment

## Sample Claim 1 Items



## Grade 3 Mathematics Item Specification C1 TB

<p><b>TM1a (DOK 1)</b> Use the Commutative Property of Multiplication to select the equivalent expression.</p> <p><b>Stimulus:</b> a whole number, <math>a \times b</math> multiplication expression</p> <p><b>Example Stem (VE):</b> Which expression is equal to <math>7 \times 4</math>?</p> <p>A. <math>4 + 7</math>            B. <math>7 - 4</math>            C. <math>4 \times 7</math>            D. <math>7 \div 4</math></p> <p><b>Rubric:</b> (1 point) The student identifies the correct expression (e.g., C).</p> <p><b>Answer Choices:</b> The answer choices should be expressions. Distractors will include common errors students make when using the properties of operations to multiply, such as breaking both factors into smaller groups and using the wrong operation (e.g., <math>7 + 4</math> or <math>7 \div 4</math>) and using another property of multiplication incorrectly (e.g., <math>4 \times 2 + 5</math>).</p> <p><b>Response Type:</b> Multiple Choice, Single Correct</p>	<p><b>TM1b (DOK 1)</b> Use the Associative Property of Multiplication to select the equivalent expression.</p> <p><b>Stimulus:</b> a whole number, <math>(a \times b) \times c</math> or <math>a \times (b \times c)</math> multiplication expression.</p> <p><b>Example Stem (M):</b> Which expression is equal to <math>2 \times (4 \times 3)</math>?</p> <p>A. <math>2 + (4 \times 3)</math>            B. <math>(2 \times 4) \times 3</math>            C. <math>(2 \times 4) + (2 \times 3)</math>            D. <math>(4 \times 3) \times (4 \times 3)</math></p> <p><b>Rubric:</b> (1 point) The student identifies the correct expression (e.g., B).</p> <p><b>Answer Choices:</b> The answer choices should be expressions. Distractors will include common errors students make when using the properties of operations to multiply, such as breaking both factors into smaller groups and using the wrong operation [e.g., <math>2 \times (4 + 3)</math>], misusing the Distributive Property of Multiplication [e.g., <math>(2 \times 4) + (2 \times 3)</math>], etc.</p> <p><b>Response Type:</b> Multiple Choice, Single Correct</p>	<p><b>TM1c-1(DOK 1)</b> Use the Distributive Property to select the equivalent expression.</p> <p><b>Stimulus:</b> a whole number, <math>a \times b</math> multiplication expression.</p> <p><b>Example Stem (H):</b> Which expression is equal to <math>5 \times 14</math>?</p> <p>A. <math>5 \times (10 + 4)</math>            B. <math>(5 \times 10) \times 4</math>            C. <math>(5 \times 1) + (2 \times 7)</math>            D. <math>(5 \times 4) \times (5 \times 10)</math></p> <p><b>Rubric:</b> (1 point) The student identifies the correct expression (e.g., A).</p> <p><b>Answer Choices:</b> The answer choices should be expressions. Distractors will include common errors students make when using the properties of operations to multiply, such as breaking both factors into smaller groups and using the wrong operation.</p> <p><b>Response Type:</b> Multiple Choice, Single Correct</p>	<p><b>TM1c-2 (DOK 1)</b> Use the Distributive Property to select the equivalent expression.</p> <p><b>Stimulus:</b> a whole number, <math>a \times (b + c)</math> or <math>a \times (b \times c)</math> multiplication expression.</p> <p><b>Example Stem (H):</b> Which expression is equal to <math>8 \times (4 + 3)</math>?</p> <p>A. <math>8 + (4 \times 3)</math>            B. <math>(8 \times 4) + 3</math>            C. <math>(8 \times 3) \times (8 \times 4)</math>            D. <math>(8 \times 4) + (8 \times 3)</math></p> <p><b>Rubric:</b> (1 point) The student identifies the correct expression (e.g., D).</p> <p><b>Answer Choices:</b> The answer choices should be expressions. Distractors will include common errors students make when using the properties of operations to multiply, such as breaking both factors into smaller groups and using the wrong operation [e.g., <math>(5 \times 3) + (3 \times 4)</math>] or [e.g., <math>(5 + 3) + (3 + 4)</math>].</p> <p><b>Response Type:</b> Multiple Choice, Single Correct</p>
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# Grade 4

## Claim 1 – Target E

579



Drag one number into each box to complete the subtraction problem shown.

- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9

Delete

$$\begin{array}{r} 50\boxed{6} \\ - \boxed{4}8\boxed{8} \\ \hline 16\boxed{8} \end{array}$$

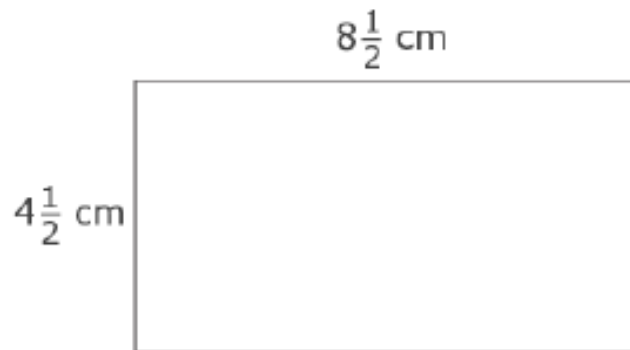
# Grade 5

## Claim 1 – Target F

622



Look at the rectangle.



What is the area of the rectangle in square centimeters?

←	→	↶	↷	✕			
1	2	3	+	-	×	÷	
4	5	6	<	≤	=	≥	>
7	8	9	$\frac{\square}{\square}$	$\square^{\square}$	( )		
0	.						

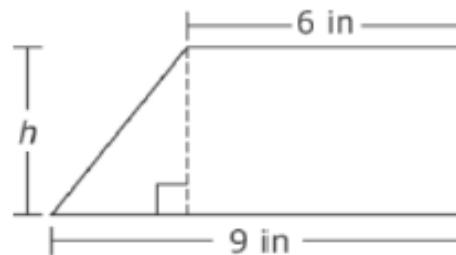
# Grade 6

## Claim 1 – Target H

631



The trapezoid shown is divided into a right triangle and a rectangle.



Use the Equation Tool to create an expression that could be used to determine the area of the trapezoid.

← → ↶ ↷ ✖

1	2	3	$h$
4	5	6	$+$ $-$ $\times$ $\div$
7	8	9	$<$ $\leq$ $=$ $\geq$ $>$
0	.	-	$\frac{\square}{\square}$ $\square^\square$ $()$ $  $

# Grade 7

## Claim 1 – Target A

734



Peter owns a lawn-mowing service. For every 3 hours of lawn-mowing, Peter charges \$28.80.

Create an equation that models the relationship between the total charge,  $y$ , and the number of hours,  $h$ , worked.

←	→	↶	↷	✖			
1	2	3	y	h			
4	5	6	+	-	×	÷	
7	8	9	<	≤	=	≥	>
0	.	-	$\frac{\square}{\square}$	$\square^\square$	( )		$\pi$

# Grade 8

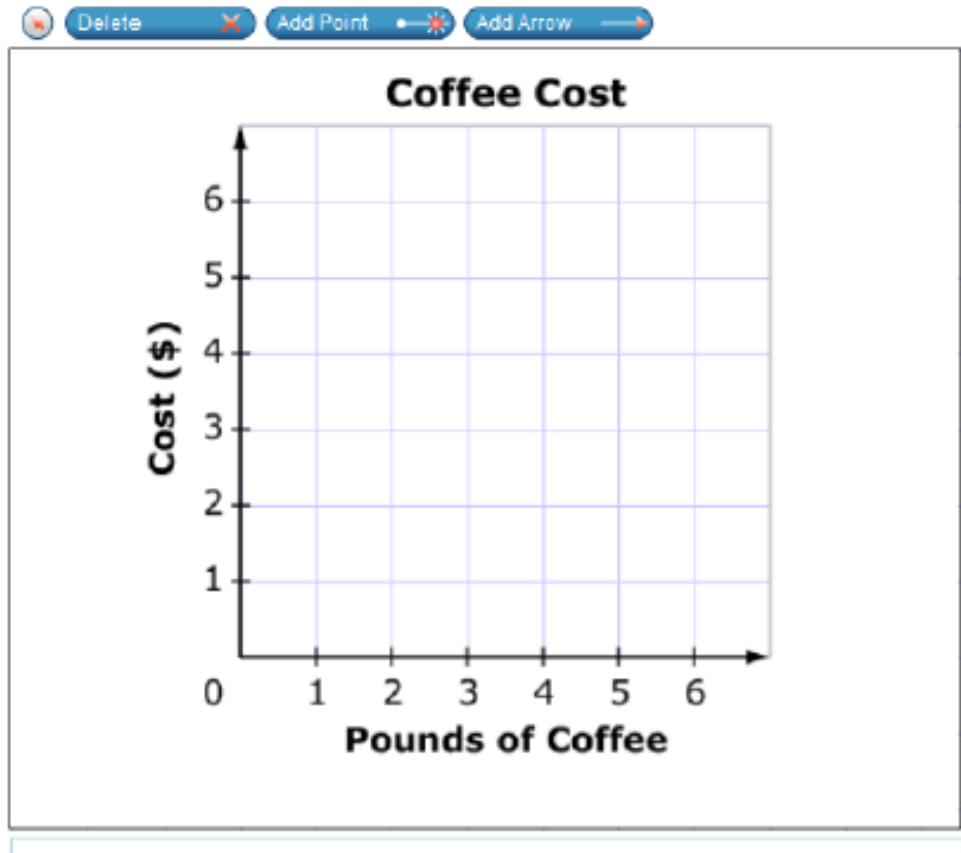
## Claim 1 - Target C

766



Coffee costs \$2.00 per pound at a coffee shop.

Use the Add Arrow tool to draw a line that shows the proportional relationship between the number of pounds of coffee purchased and the total cost.



# Grade 11

## Claim 1 – Target E (Algebra)

666



Consider the function

$$f(x) = x^2 - 5x - 14.$$

Which of the numbers in the chart are zeros of the function?  
Select Yes or No in each row.

$$f(x) = x^2 - 5x - 14$$

Is this a zero of the function?	Yes	No
2		
7		
-2		
-7		



# Item Quality Criteria

- 1a. Does the item provide evidence to support the intended claim? Does the item closely align to the claim, target and primary Common Core standard (including cluster level)?
- 1b. Is this the most appropriate item type to gather evidence to support the target and standard?
- 1c. Is the item mathematically correct, including its use of precise mathematical language?
- 1d. Is the item worth asking?
- 1e. Does the item appear to be accessible to all students? If not, could the item be revised to be made more accessible and still measure the target and standard?
- 1f. Do the answer choices or rubrics capture the essence of the target and standard?
- 1g. Is the item/task developmentally appropriate?

# Accessibility and Accommodations



# Accessibility and Accommodations Considerations for Claim 1

- Visual graphics may be difficult or inaccessible for students who are blind or visually impaired. The simplest graphics should be used to minimize this issue.
- Students with dyscalculia may have difficulty with the calculations.
- All vocabulary should be at or below grade level to minimize reading load.
- Drag and Drop response types may not be accessible for students who are visually impaired. Alternate item response types are available.
- Task Models will be identified as more appropriate for populations with various disabilities

# Questions



# Reflect on Guiding Questions

- What do educators need to do to support student learning?
- What do educators need to do to ensure that students are prepared for the Smarter Balanced assessments?