Pacing: 5 weeks (plus 1 week for re-teaching/enrichment)

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| **Mathematical Practices** |
| *Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning.*  *Practices in bold are to be emphasized in the unit.*  **1. Make sense of problems and persevere in solving them.**  2. Reason abstractly and quantitatively.  **3. Construct viable arguments and critique the reasoning of others.**  **4. Model with mathematics.**  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |
| **Domain and Standards Overview** |
| **Statistics and Probability**   * Develop understanding of statistical variability. * Summarize and describe distributions. |

| **Priority and** Supporting **CCSS** | **Explanations and Examples\*** |
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| **6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.**  6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.*  6.SP.2.Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. | **6.SP.3.** When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values.  Example:  • Consider the data shown in the dot plot of the six trait scores for organization for a group of students.  o How many students are represented in the data set?  o What are the mean, median, and mode of the data set? What do these values mean? How do they compare?  o What is the range of the data? What does this value mean?    6.SP.1. Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents).  Questions can result in a narrow or wide range of numerical values. For example, asking classmates “How old are the students in my class in years?” will result in less variability than asking “How old are the students in my class in months?”  Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: “How many hours per week on average do students at Jefferson Middle School exercise?”  To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers.  6.SP.2.  The two dot plots show the 6-trait writing scores for a group of students on two different traits, organization and ideas. The center, spread and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry. Showing the two graphs vertically rather than side by side helps students make comparisons.  For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5. |
| **6.SP.5. Summarize numerical data sets in relation to their context, such as by:**   1. **Reporting the number of observations.** 2. **Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.** 3. **Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.**   **Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.** | 6.SP.5. Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation.  The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.  (continued on the next page)  Understanding the Mean  The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair.  The leveling process can be connected to and used to develop understanding of the computation of the mean. For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or stacking cubes.  Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters.  This data set could be represented with stacking cubes.  Students can model the mean by “leveling” the stacks or distributing the blocks so the stacks are “fair”. Students are seeking to answer the question “If all of the students had the same number of letters in their name, how many letters would each person have?”  One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.  If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have.  Understanding Mean Absolute Deviation  The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.  (Continued on next page)  In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.  To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.    The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be 6 ÷ 8 or ¾ or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set.  Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set? (continued on the next page)  The mean of this data set is still 5.  Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set?  The mean of this data set is still 5.  The mean deviation of this data set is 16 ÷ 8 or 2. Although the mean is the same, there is much more variability in this data set.  (Continued on next page)  Understanding Medians and Quartiles  Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles (Q3 – Q1). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed.  Consider the first data set again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest.  The middle value in the ordered data set is the median. If there are an even number of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4th and 5th values which are both 5. Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values which is an even number of values. Q1 would be the average of the 2nd and 3rd value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6th and 7th value in the data set or 5.5. The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 (5.5 – 4). The interquartile range is small, showing little variability in the data. |
| 6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. | 6.SP.4. In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM’s Illuminations.  Box Plot Tool - [http://illuminations.nctm.org/ActivityDetail.aspx?ID=77](http://illuminations.nctm.org/ActivityDetail.aspx?ID=77%20)  Histogram Tool -- [http://illuminations.nctm.org/ActivityDetail.aspx?ID=78](http://illuminations.nctm.org/ActivityDetail.aspx?ID=78%20)  Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.  In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students bin the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions you may draw from it.  Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summary of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data.  Examples:  • Nineteen students completed a writing sample that was scored using the six traits rubric. The scores for the trait of organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?  (Continued on next page)  • Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?    A histogram using 5 bins (0-9, 10-19, …30-39) to organize the data is displayed below.  • Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?  **Five number summary**  Minimum – 130 months  Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months  Median (Q2) – 139 months  Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months  Maximum – 150 months  (Continued on next page)    This box plot shows that  • ¼ of the students in the class are from 130 to 132.5 months old  • ¼ of the students in the class are from 142.5 months to 150 months old  • ½ of the class are from 132.5 to 142.5 months old  • the median class age is 139 months. |

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| **Concepts**  **What Students Need to Know** | **Skills**  **What Students Need To Be Able To Do** | **Bloom’s Taxonomy Levels** |
| * Measure of center   + Median   + Mean * Measure of variation   + Range   + Interquartile range   + Mean absolute deviation * Numerical data sets   + Observations   + Attributes   + Overall pattern     - Deviations from   + Choice of measures of center and variability   + Shape of the data distribution     - Center     - Spread     - Variability * Statistical question * Numerical data displays   + Number line   + Dot plot   + Histogram   + Box plot | * RECOGNIZE (measure of center and measure of variation) * SUMMARIZE (numerical data sets) * REPORT (observations) * DESCRIBE (attribute) * GIVE/FIND (measure of center and measure of variation) * DESCRIBE (overall pattern) * RELATE (choice of measure to shape of the data) * RECOGNIZE (a statistical question) * UNDERSTAND (data distribution is described by its center, spread, and overall shape) * Display (numerical data) | 2  4  1  1  3  4  4,5  2  2  3 |

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| **Essential Questions** |
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| **Corresponding Big Ideas** |
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| **Standardized Assessment Correlations**  **(State, College and Career)** |
| **Expectations for Learning (in development)**  This information will be included as it is developed at the national level. CT is a governing member of the Smarter Balanced Assessment Consortium (SBAC) and has input into the development of the assessment. |

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| **Tasks and Lessons from the Mathematics Assessment Project (Shell Center/MARS, University of Nottingham & UC Berkeley)**  **These tasks can be used during the course of instruction when deemed appropriate by the teacher.** |
| **Suzi’s Company** <http://map.mathshell.org/materials/tasks.php?taskid=383&subpage=apprentice>  **Candy Bars** <http://map.mathshell.org/materials/tasks.php?taskid=396&subpage=expert> |

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| **Tasks from Inside Mathematics (**<http://insidemathematics.org/index.php/mathematical-content-standards>)  **These tasks can be used during the course of instruction when deemed appropriate by the teacher.**  **NOTE: Most of these tasks have a section for teacher reflection.** |
| **Baseball Players** |

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| **Unit Assessments**  **The items developed for this section can be used during the course of instruction when deemed appropriate by the teacher.** |
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