Math Practice Standards: Classroom Evidence

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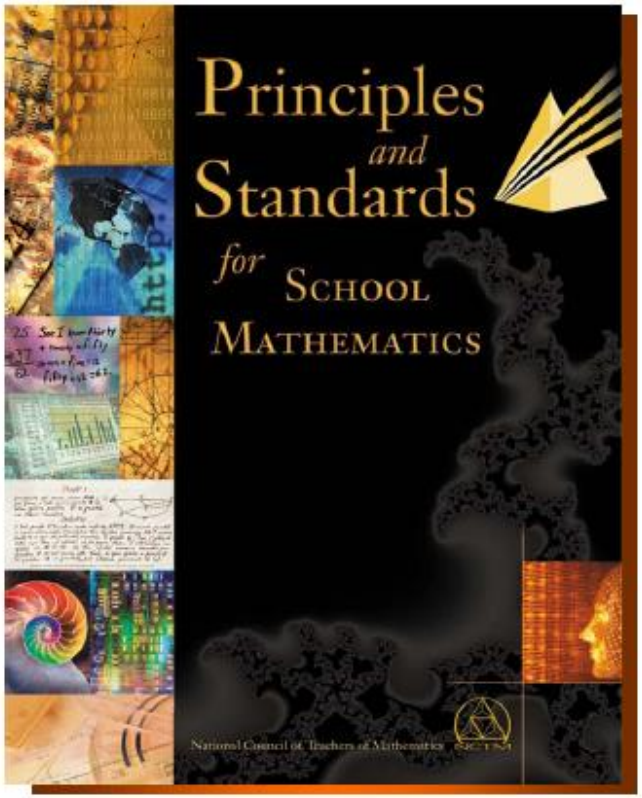
What's in the CCS-Math?

- The Standards for Mathematical Content
  - Define what students should understand and be able to do
  - K-8 Grade Level Domains
  - High School Conceptual Categories
- The Standards for Mathematical Practice
  - Describe HABITS OF MIND of a mathematically expert student
  - Recurring throughout the grades

Background of the Math Practice Standards

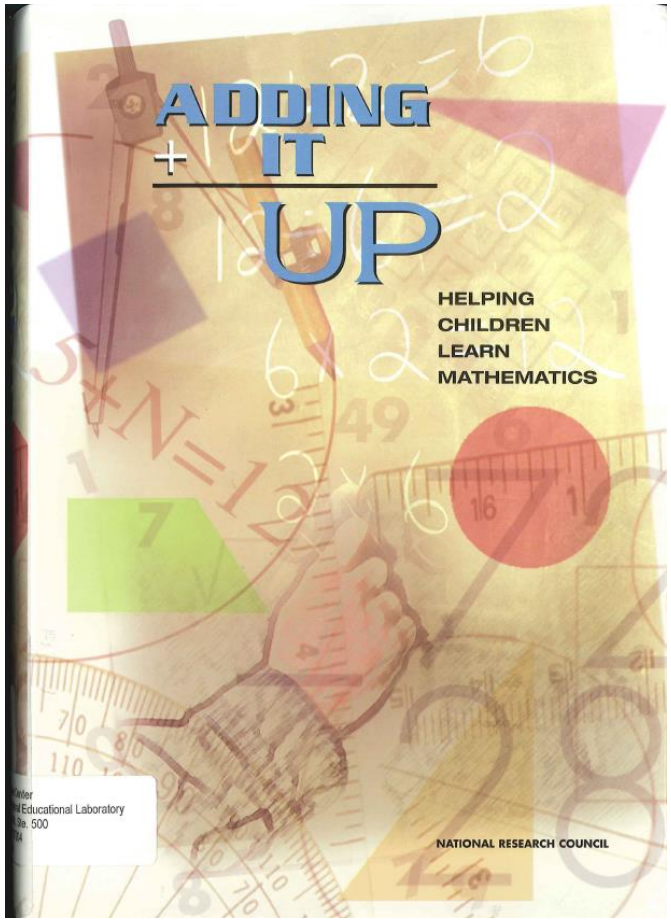
Draws from Two Sources

- *Principals and Standards for School Mathematics*



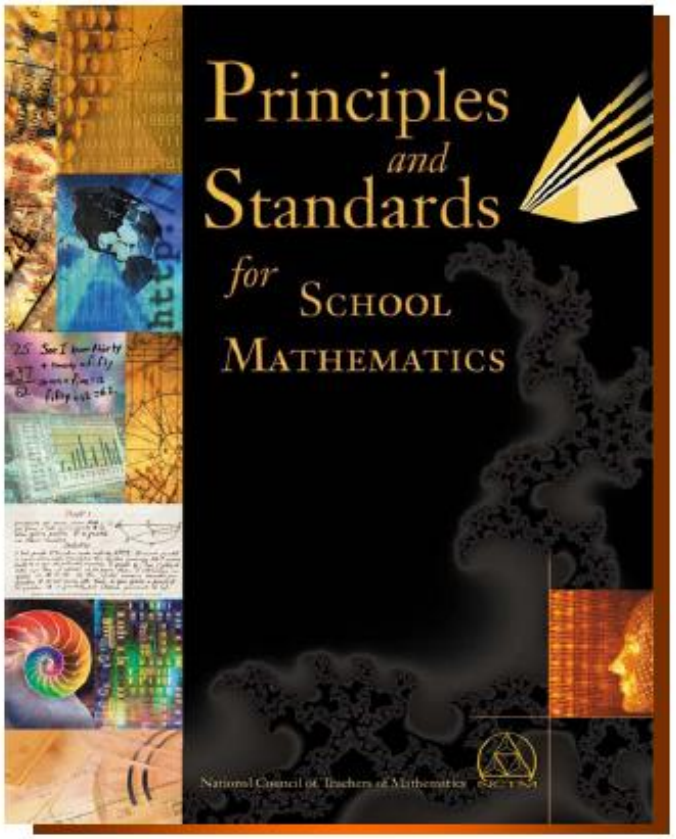
NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

- *Adding It Up*



NCTM Principles and Standards for School Mathematics

- Problem Solving
- Reasoning and Proof
- Communication
- Representation
- Connections



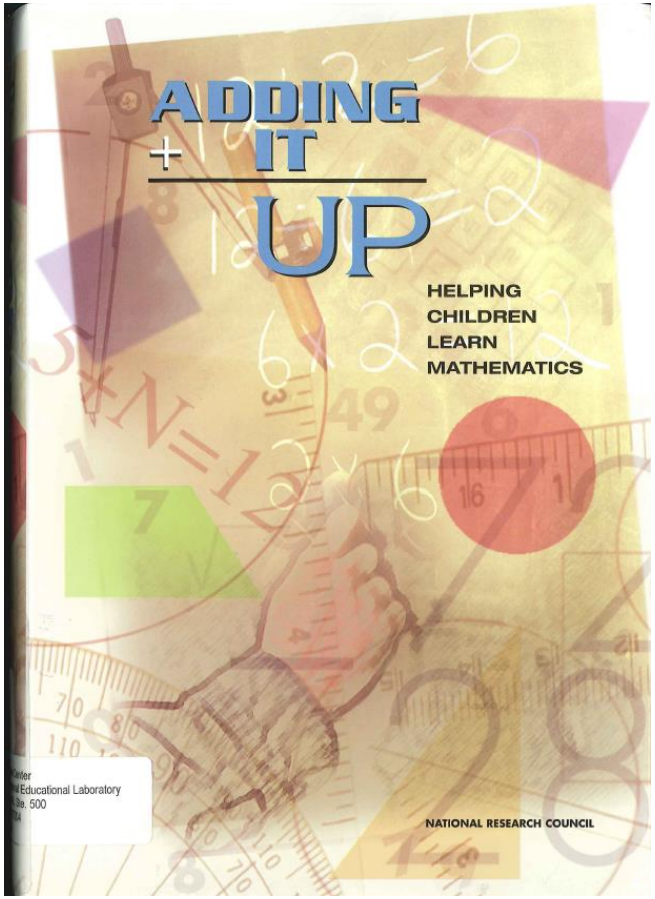
NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

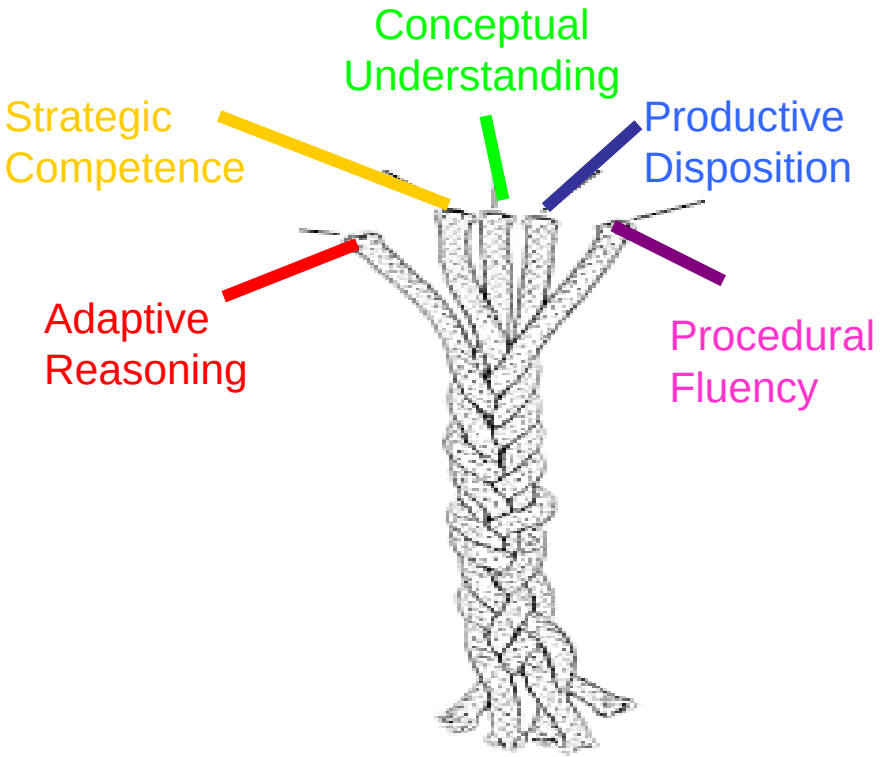
Process Standards

- ***Problem Solving -*** The process of applying a variety of appropriate strategies based on information provided, referenced, recalled, or developed.
- ***Reasoning and Proof –*** Making and investigating mathematical conjectures. Developing arguments and proofs.
- ***Communication –*** Organizing mathematical thinking coherently and clearly to peers, teachers and others. Using the language of math to express mathematical ideas precisely.
- ***Representation –*** Creating and using multiple representations to organize, record and communicate mathematical ideas. Using models and interpreting mathematical phenomena.
- ***Connections –*** Recognizing and using connections among math ideas as well as other subjects. Understanding how mathematical ideas interconnect and build on one another.

https://mathequality.wordpress.com/2012/06/26/nctm-process-standards-for-mathematics/

Adding It Up: Strands of Mathematical Proficiency





Strands of Mathematical Proficiency

- ***Conceptual Understanding*** – comprehension of mathematical concepts, operations, and relations
- ***Procedural Fluency*** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic Competence*** – ability to formulate, represent, and solve mathematical problems
- ***Adaptive Reasoning*** – capacity for logical thought, reflection, explanation, and justification
- ***Productive Disposition*** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Crosswalk

NCTM Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Representation
- Connections

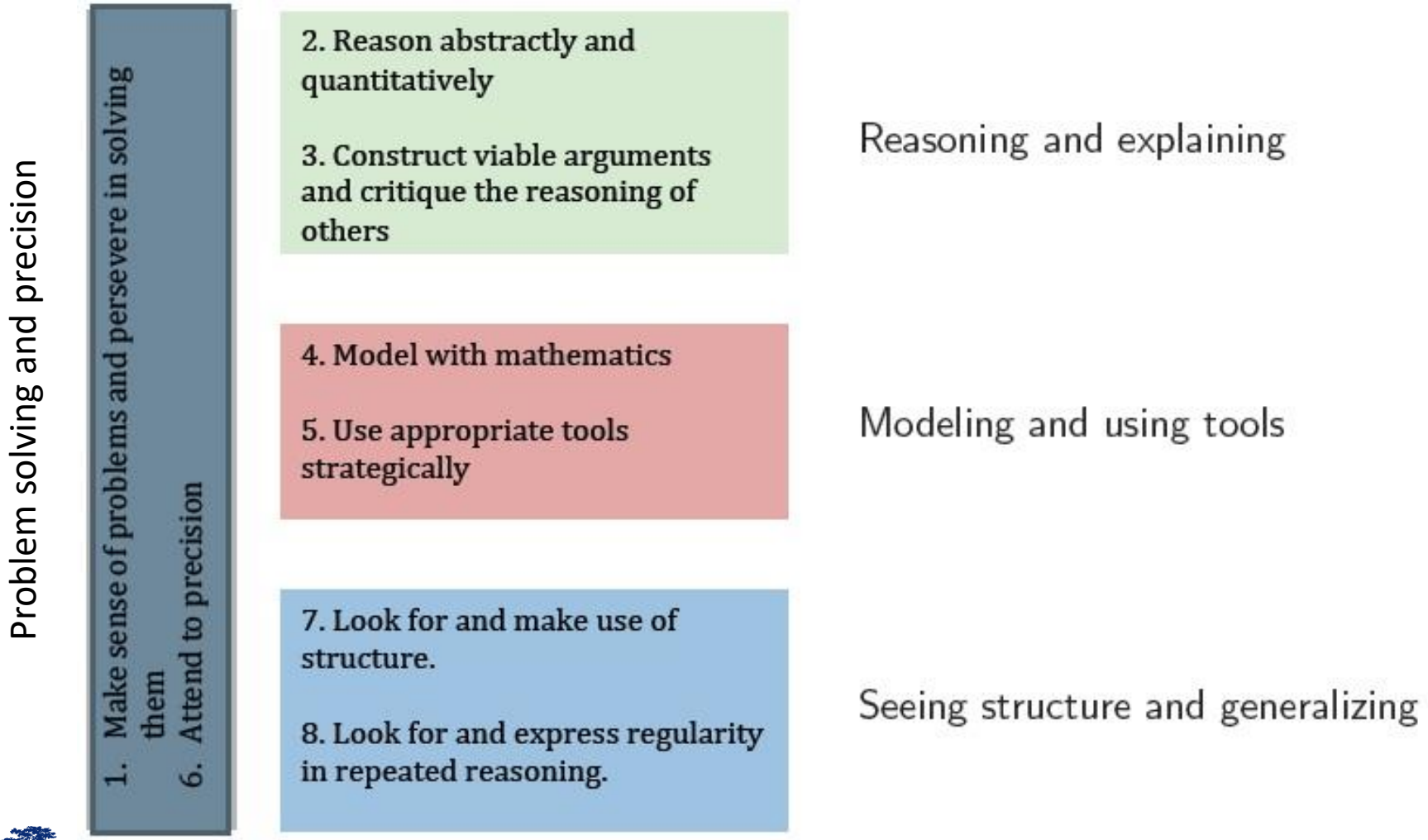
Strands of Mathematical Proficiency

- Conceptual Understanding
- Procedural Fluency
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition

The Standards for Mathematical Practice

1. *Make sense of problems and persevere in solving them.*
2. *Reason abstractly and quantitatively.*
3. *Construct viable arguments and critique the reasoning of others.*
4. *Model with mathematics.*
5. *Use appropriate tools strategically.*
6. *Attend to precision.*
7. *Look for and make use of structure.*
8. *Look for and express regularity in repeated reasoning.*

Grouping the Standards



Bill McCallum, 2011

Understanding the Math Practice Standards

Standards for Mathematical Practice Dual Nature

Standards for Mathematical Practice describe *mathematical content* students need to learn.

SP1. Make sense of problems

"… students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends."

Standards for Mathematical Practice describe the *nature of the learning experiences, thinking processes, habits of mind, and dispositions* that students need to develop a deep, flexible, and enduring understanding of mathematics.

SP1. Make sense of problems

"….they [students] analyze givens, constraints, relationships and goals. ….they monitor and evaluate their progress and change course if necessary. …. and they continually ask themselves "Does this make sense?"

The Beginning of the Practice Standards

Take a moment to examine the first three words of each of the 8 mathematical practices.

What do you notice?

Mathematically Proficient Students…

Practice Standards Actions

What are the *verbs* that illustrate the student actions for your assigned mathematical practice?

Circle, highlight or underline them for each practice.

Discuss with a partner:

What jumps out at you?

What few words summarize the actions needed for the practice?

What implications might each standard for mathematical practice have on classroom instruction?

The Standards for Mathematical Practice

SMP1: *Explain and make conjectures…*

SMP2: *Make sense of…*

SMP3: *Understand and use…*

SMP4: *Apply and interpret…*

SMP5: *Consider and detect…*

SMP6: *Communicate precisely to others…*

SMP7: *Discern and recognize…*

SMP8: *Notice and pay attention to…*

An Illustration of the Practices

Activity 4

Deep Dive Into Implementation

Successfully Implementing the Standards …..

Requires:

- Teacher knowledge of mathematics language and content.
- Teacher knowledge of how to promote student involvement in mathematical practices.
- Shifting students' focus from "answer getting" to solving problems.
- Establishing the classroom environment as a community of learners.

Implementing the Practices

- **Content is the vehicle for engaging in the Standards for Mathematical Practice.**
  - ✓ Practices should be embedded in classroom instruction, discussions, activities and assessment and connected to content in meaningful ways.

- **The Standards for Mathematical Practice are not a checklist.**
  - ✓ Practices work together and a single student behavior could be thought of as exhibiting multiple practices at once.

- **The Standards for Mathematical Practice look different across the grades as students grow in mathematical maturity.**
  - ✓ There is a progression of the math practice standards.

Lessons

Tasks

- Engage the students in the math practice standards
- Contain elements that speak to specific math practices

Teacher Actions

- Develops the math practice standards within the students

Questioning

- Effective questioning can support the student's development of the mathematical practices
- Effective questioning invites students to explain their thinking, make new connections, describe their process, or critique other ideas
- Questioning should be aligned to the mathematical practice

What do classrooms look like if engaged in the math practices?

Task Element MP1

MP 1: Make sense of problems and persevere in solving them

- Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- Requires cognitive effort - while procedures may be followed, the approach or pathway is not explicitly suggested by the task, or task instructions and multiple entry points are available.
- Encourages multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

https://achievethecore.org/aligned/wp-content/uploads/2016/06/Implementing-Standards-for-Mathematical-Practices-Updated-2016.pdf

Teacher Actions MP1

MP 1: Make sense of problems and persevere in solving them

- Allows students time to initiate a plan; uses question prompts as needed to assist students in developing a pathway.
- Continually asks students if their plans and solutions make sense.
- Questions students to see connections to previous solution attempts and/or tasks to make sense of current problem.
- Consistently asks to defend and justify their solution by comparing solution paths.

https://achievethecore.org/aligned/wp-content/uploads/2016/06/Implementing-Standards-for-Mathematical-Practices-Updated-2016.pdf

In Practice: MP1

Activity 5

Task Element MP2

MP 2: Reason Abstractly and quanitatively

- Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- Consistently expects students to convert situations into symbols in order to solve the problem; and then requires students to explain the solution within a meaningful situation.
- Contains relevant, realistic content.

https://achievethecore.org/aligned/wp-content/uploads/2016/06/Implementing-Standards-for-Mathematical-Practices-Updated-2016.pdf

Teacher Actions MP2

MP 2: Reason Abstractly and quantitatively

- Asks students to explain the meaning of the symbols in the problem and in their solution.
- Expects students to give meaning to all quantities in the task.
- Questions students so that understanding of the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

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In Practice: MP2

Task Element MP3

MP 3: Construct viable arguments and critique the reasoning of others

- Is structured to bring out multiple representations, approaches, or error analysis.
- Embeds discussion and communication of reasoning and justification with others.
- Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- Expects students to give feedback and ask questions of others' solutions.

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Task 1

Joel and Marisa are running for president at their middle school (grades 6-8). After the votes are in, Joel and Marisa are each convinced that they have won the election.

- Joel argues that he has won a larger percentage of the overall vote than Marisa so he should be the new president.
- Marisa argues that she has won a larger percentage than Joel of the 6th grade vote and the 7th grade vote. Since the majority of the grades voted for her, she should be the new president.

Is it possible that both Joel and Marisa are making accurate claims? Explain.

Teacher Actions MP3

MP 3: Construct viable arguments and critique the reasoning of others

- Encourages students to use proven mathematical understandings, (definitions, properties, conventions, theorems etc.), to support their reasoning.
- Questions students so they can tell the difference between assumptions and logical conjectures.
- Asks questions that require students to justify their solution and their solution pathway.
- Prompts students to respectfully evaluate peer arguments when solutions are shared.
- Asks students to compare and contrast various solution methods
- Creates various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.)

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In Practice: MP3

Task Element MP4

MP 4: Model with mathematics

- Is structured so that students represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.
- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.

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Teacher Actions MP4

MP 4: Model with mathematics

- Demonstrates and provides students experiences with the use of various mathematical models.
- Questions students to justify their choice of model and the thinking behind the model.
- Asks students about the appropriateness of the model chosen.
- Assists students in seeing and making connections among models.

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In Practice: MP4

Task Element MP5

MP 5: Use appropriate tools strategically

- Lends itself to multiple learning tools. (Tools may include; concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- Requires students to determine and use appropriate tools to solve problems.
- Asks students to estimate in a variety of situations:
  - a task when there is no need to have an exact answer
  - a task when there is not enough information to get an exact answer
  - a task to check if the answer from a calculation is reasonable

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Teacher Actions MP5

MP 5: Use appropriate tools strategically

- Demonstrates and provides students experiences with the use of various math tools. A variety of tools are within the environment and readily available.
- Questions students as to why they chose the tools they used to solve the problem.
- Consistently models how and when to estimate effectively, and requires students to use estimation strategies in a variety of situations.
- Asks student to explain their mathematical thinking with the chosen tool.
- Asks students to explore other options when some tools are not available.

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In Practice: MP5

Task Element MP6

MP 6: Attend to precision

- Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- Expects students to use symbols appropriately.
- Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

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Teacher Actions MP6

MP 6: Attend to precision

- Consistently uses and models correct content terminology.
- Expects students to use precise mathematical vocabulary during mathematical conversations.
- Questions students to identify symbols, quantities and units in a clear manner.

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In Practice: MP6

Task Element MP7

MP 7: Look for and make use of structure.

- Requires students to look for the structure within mathematics in order to solve the problem. (i.e. – decomposing numbers by place value; working with properties; etc.)
- Asks students to take a complex idea and then identify and use the component parts to solve problems. i.e. Building on the structure of equal sharing, students connect the understanding to the traditional division algorithm. When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size"such as with long division
- Expects students to recognize and identify structures from previous experience(s) and apply this understanding in a new situation. i.e. 7 x 8 = (7 x 5) + (7 x 3) OR 7 x 8 = (7 x 4) + (7 x 4)…new situations could be, distributive property, area of composite figures, multiplication fact strategies. .

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Teacher Actions MP7

MP 7: Look for and make use of structure.

- Encourages students to look for something they recognize and have students apply the information in identifying solution paths (i.e. composing/decomposing numbers and geometric figures, identifying properties, operations, etc.)
- Expects students to explain the overall structure of the problem and the big math idea used to solve the problem.

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In Practice: MP7

Task Element MP8

MP 8: Look for and express regularity in repeated reasoning.

- Present several opportunities to reveal patterns or repetition in thinking, so students can make a generalization or rule.
- Requires students to see patterns or relationships in order to develop a mathematical rule.
- Expects students to discover the underlying structure of the problem and come to a generalization.
- Connects to a previous task to extend learning of a mathematical concept.

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Task 2

Track your thinking so you can explain your thinking to someone else. In all cases, try to NOT calculate; rather REASON about the two quantities.

1. Which quantity is greater?

i.	694 – 345 + 233	233 – 345 + 694
ii.	790 + 135 + 232	795 + 133 + 230
iii.	13 – 9 + 12 – 11	0
iv.	$\frac{1}{6}(3 * 6)$	1
v.	$3(x - 7) - 2(x - 7)$	$x - 7$
vi.	$4 + (x + 3)^2$	π
vii.	5% of 7 billion	7% of 5 billion

Teacher Actions MP8

MP 8: Look for and express regularity in repeated reasoning.

- Asks what math relationships or patterns can be used to assist in making sense of the problem.
- Asks for predictions about solutions at midpoints throughout the solution process.
- Questions students to assist them in creating generalizations based on repetition in thinking and procedures.

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In Practice: MP8

Summary of Evidence

- Students are doing the work, not merely mimicking the work the teacher demonstrated.
- Student work products include more than just calculations.
  - Claims and justifications
  - Models and representations
  - Classifications
- Students ask questions that they're not sure anybody knows the answer to.

Assessing the Math Practice Standards

The Link to Assessment

- For Claims 2, 3, and 4 on the Smarter Balanced Assessment, the assessment targets are drawn from the CT Core Standards for Mathematical Practice.
- The assessment targets for Claims 2, 3, and 4 are the same across all tested grades.

Grade 4 Sample Assessment Item

Pablo solved a multiplication problem using two different methods. He made a mistake in either Method W or Method Z.

Method W

23 × 49

20 × 9 = 180
3 × 9 = 27
20 × 4 = 80
3 × 4 = + 12
299

Method Z

23 × 49

Area Model

	40	+ 9
20	800	180
+ 3	120	27

Rectangle Sections

1
800
120
180
+ 27
1,127

Identify the method where Pablo made a mistake and explain what he should do to correct it. Type your answer in the box below.

Finding Evidence

Core Action 3

Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson

RATING:

4 – Teacher provides many opportunities, and most students take them
3 – Teacher provides many opportunities, and some students take them **or**
Teacher provides some opportunities and most students take them
2 – Teacher provides some opportunities, and some students take them
1 – Teacher provides few or no opportunities **or**
few or very few students take the opportunities provided

Core Action 3 Teacher Indicators

Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

A. The teacher provides opportunities for all students to work with and practice course-level problems and exercises.

B. The teacher cultivates reasoning and problem solving by allowing students to productively struggle.

C. The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson.

D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking.

E. The teacher connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas.

Core Action 3 Student Indicators

Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

A. Students work with and practice course-level problems and exercises.

B. Students persevere in solving problems in the face of difficulty.

C. Students share their thinking about the content of the lesson beyond just stating answers.

D. Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.

E. Students use increasingly precise mathematical language and ideas.

Activity 8

What instructional decisions did the teacher make that seemed to support the development of Standards for Mathematical Practice for students?

What did you hear or see in a mathematics classroom that is working to build the Standards for Mathematical Practice?

Standards for Mathematical Practice

The eight standards for mathematical practice place an emphasis on students *doing* mathematics and *demonstrating* learning.

Equitable achievement will begin with an understanding of how the
- *selection of tasks,*
- *assessment of tasks, and*
- *student learning environment*

can support or undermine equity in our schools.

Final Thought

How do I know that all students have opportunities to exhibit mathematical practices while engaging with the content of the lesson?

- Are questions and problems posed that prompt students to share their developing thinking?
- What opportunities do students have to work with and practice grade level problems?
- What strategies are used to encourage collaboration among students?
- Do students explain their thinking?
- Do students talk about each other's thinking?
- Do students persist in solving challenging problems?
- Does the teacher connect student's informal language to precise mathematical language?


QUESTIONS

Thank You

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