The Shifts for CCS – Math

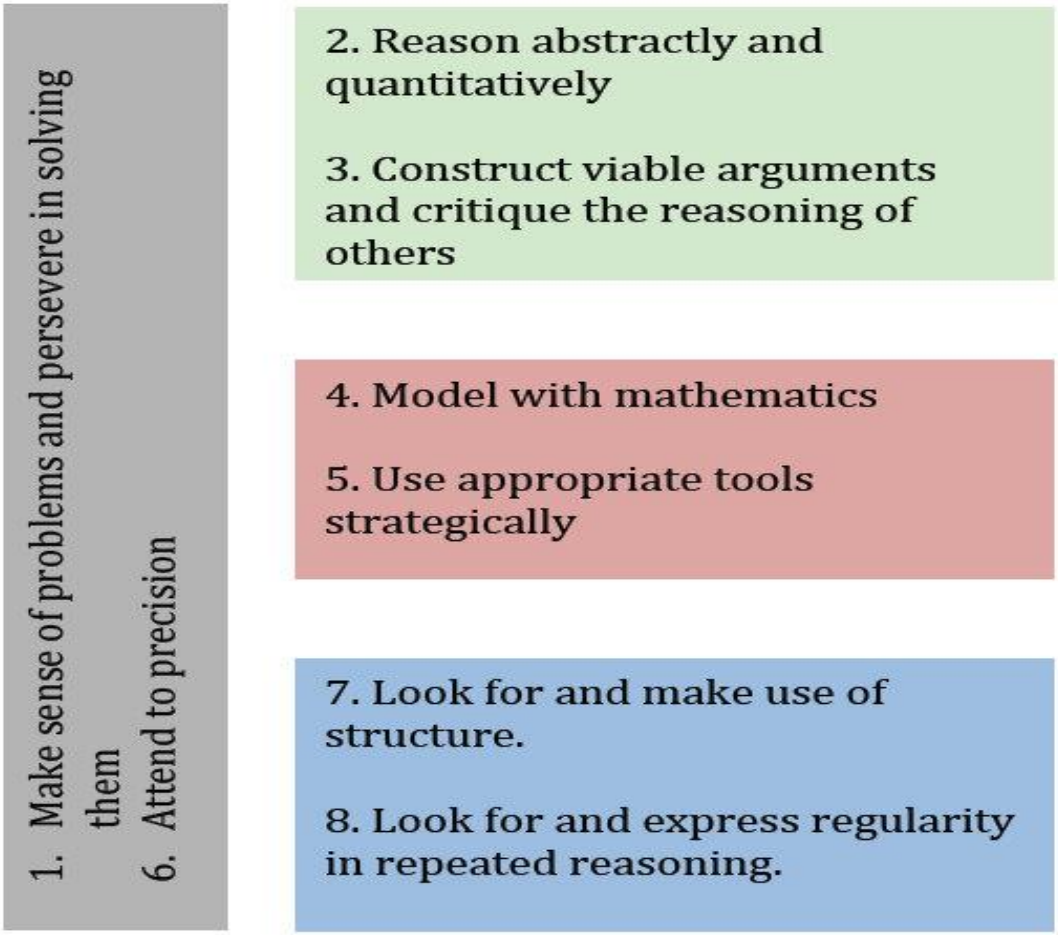
A Webinar Series

A Look Into The Standards

What's in the CCS-Math?

- The Standards for Mathematical Content
  - What students should know and be able to do
  - Content Domains
  - Domains into clusters
  - Clusters into standards
- The Standards for Mathematical Practice
  - The habits of mind

Organization of the Standards for Mathematical Practice



The *Standards for Mathematical Practice* were based on the NCTM Process Standards and the National Research Council's (NRC) Strands of Mathematical Proficiency.

Three Instructional Shifts for CCS - Mathematics

Focus on the Standards; teach less but for understanding.

Coherence – Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.

Rigor – Means a balance of solid conceptual understanding, procedural skill and fluency, and application of skills in problem solving situations.

Coherence

What are learning progressions?

- "descriptions of the successively more sophisticated ways of thinking about an idea that follow one another as students learn" (Wilson & Bertenthal, 2005)
- "a picture of the path students typically follow as they learn…a description of skills, understandings, and knowledge in the sequence in which they typically develop" (Masters & Forster, 1996)
- "a sequenced set of subskills and bodies of enabling knowledge that, it is believed, students must master en route to mastering a more remote curricular aim" (Popham, 2008)
- "a description of how student understanding or learning can or should develop over time" (Gong, 2008)

Why Coherence?

- Mathematics is not a list of disconnected topics, tricks, or mnemonics;

- Mathematics is a coherent body of knowledge made up of interconnected concepts.

Mathematics Progressions:
The beginning of the standards

- Narrative documents describing the progression of a topic across a number of grade levels
- Informed both by educational research and the structure of mathematics
- Sliced into grade level standards

The Common Core Standards Writing Team

Why Coherence?

- The standards are designed around coherent progressions from grade to grade
- Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years.

Benefits of Using the Progressions

With a possible learning path in mind educators can…

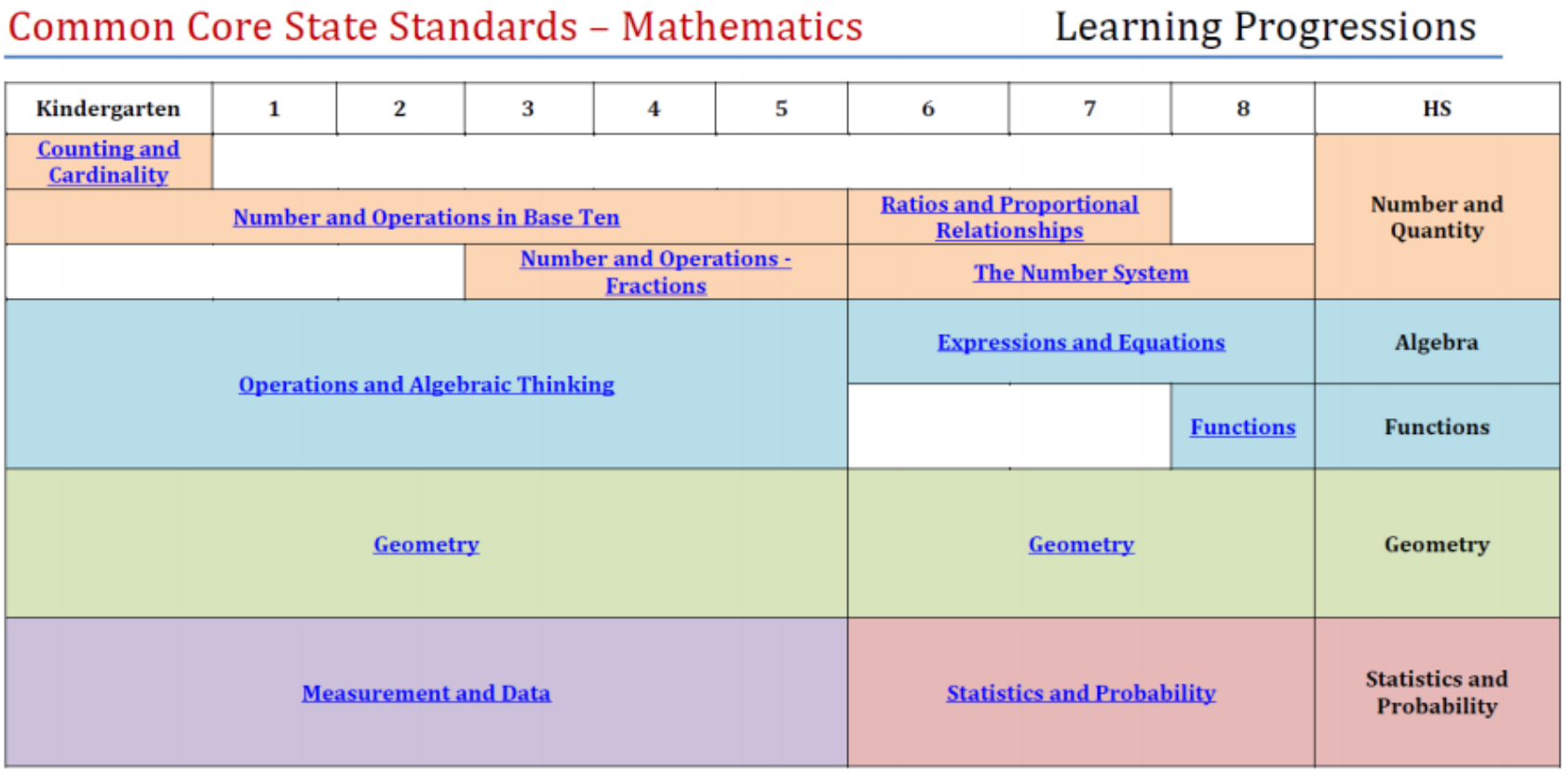
- Consider strategies for instructional scaffolding to get students to the next stage of learning
- Use formative & summative assessments “strategically” and more frequently; they value “uncovering student thinking”
- Collaboratively analyze student work creating a deeper understanding of how learning develops
- Adjust instruction according to what students CAN do, not what they CANNOT do
- Shift perceptions, especially of their lower performing students & what to do next to support learning

Shift 2: Coherence

Think across grades and link to major topics within grades

- ✓ Connect learning within and across grades
- ✓ Each standard is not a new event, but an extension of previous learning
- ✓ Mathematics makes sense.

Coherence:
Built on the Progressions



Ohio Department of Education (12/14/10)

Progress to Algebra in Grades K-8

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

Two Levels of Coherence

- Coherence within a grade
  - Reinforce a major topic in a grade by utilizing a supporting topic
  - Meaningful introduction to topics in the same grade that complement each other
- Coherence across grades
  - Apply learning from previous grades to learn new topics
  - Progressions of mathematics that are meaningful and make sense

Coherence Within a Grade

Example: Data Representation

1.MD.C.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, **how many in each category, and how many more or less** are in one category than in another.

Coherence Within a Grade

Example: Statistics

8.SP.A.3 **Use the equation of a linear model** to solve problems in the context of bivariate measurement data, **interpreting the slope and intercept**. *For example, in a linear model for a biology experiment, interpret a slope of 1.5cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

Coherence Within a Grade

CCSS.MATH.CONTENT.4.MD.A.2

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money…

CCSS.MATH.CONTENT.4.OA.A.1

Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Elena has a cat with a mass of 4 kilograms. Ginger's cat has a mass that is 2 times as much as Elena's cat. What is the mass of Ginger's cat in grams?

A Look Within the Conceptual Categories

Reasoning with Equations and Inequalities	***Reasoning with Equations and Inequalities***
A.REI.6: **Solve systems of linear equations** exactly and approximately (e.g., with graphs), **focusing on pairs of linear equations in two variables.**	A.REI.7: **Solve a simple system consisting of a linear equation and a quadratic equation** in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Coherence Supported By Instructional Materials

Materials must focus coherently on the Major Work of the grade in a way that is consistent with the progressions in the Standards.

Materials follow the grade-by-grade progressions in the Standards. Content from previous or future grades does not unduly interfere with or displace on-grade-level content.

Progression Evidence in Resources

BIG IDEA 1 Multiplication with Tens and Hundreds

Common Core State Standards 4.NBT.1, 4.NBT.5

1 **Arrays and Area Models**
ACTIVITIES Arrays of Ones • Arrays of Tens

2 **Connect Place Value and Multiplication**
ACTIVITIES Discuss a Product of Tens • Arrays of Hundreds

3 **Mental Math and Multiplication**
ACTIVITIES Review Multiplication with Tens • Mental Multiplication

BIG IDEA 2 Multiply by One-Digit Numbers

Common Core State Standards 4.OA.3, 4.NBT.2, 4.NBT.3, 4.NBT.5, 4.MD.2

4 **Model One-Digit by Two-Digit Multiplication**
ACTIVITIES Multiplication Modeling • Practice Multiplication • Multiplication with Dollars

5 **Estimate Products**
ACTIVITIES Estimate Products • Practice Estimation

6 **Use Place Value to Multiply**
ACTIVITIES Model the Place Value Sections Method • Model the Expanded Notation Method

7 **Algebraic Notation Method**
ACTIVITIES The Distributive Property and Multiplication • Connect Models and the Distributive Property

8 **Compare Methods of One-Digit by Two-Digit Multiplication**
ACTIVITIES Multiplication Methods • Practice Multiplication

9 **Discuss Different Methods**
ACTIVITIES Compare Multiplication Methods • Analyze the Shortcut Method

10 **One-Digit by Three-Digit Multiplication**
ACTIVITIES Multiply One-Digit Numbers by Hundreds • Use the Area Model to Multiply Hundreds • Practice One-Digit by Three-Digit Multiplication

BIG IDEA 3 Multiplication with Two-Digit Numbers

Common Core State Standards 4.OA.3, 4.NBT.2, 4.NBT.5

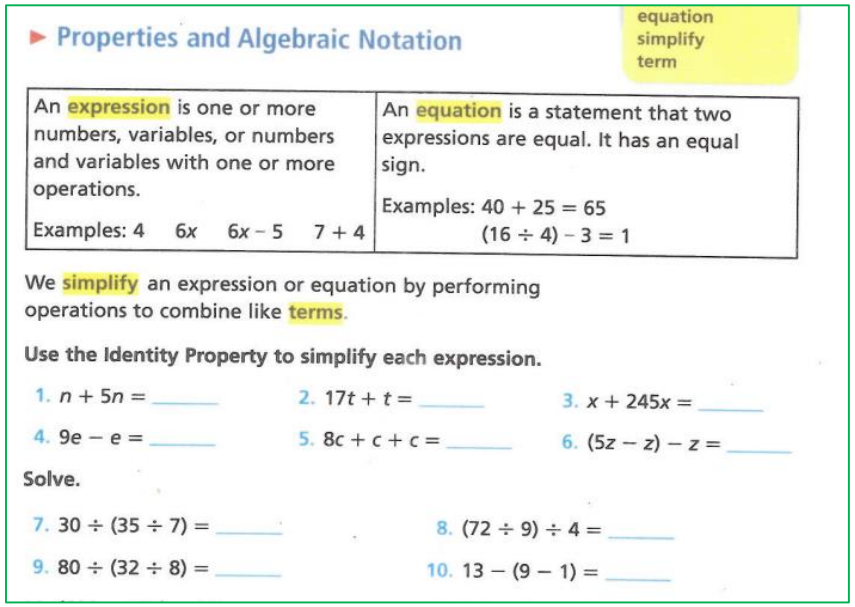
12 **Two-Digit by Two-Digit Multiplication**
ACTIVITIES Represent Multiplication • Practice Multiplication

13 **Different Methods for Two-Digit Multiplication**
ACTIVITIES Multiply Two-Digit Numbers • The Shortcut Multiplication Method

14 **Check Products of Two-Digit Numbers**
ACTIVITIES Compare Methods • Estimate Products of Two-Digit Numbers • Practice Multiplication

What about content from other grades?

4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.



► Properties and Algebraic Notation

equation
simplify
term

An expression is one or more numbers, variables, or numbers and variables with one or more operations. Examples: 4 $6x$ $6x - 5$ $7 + 4$	An equation is a statement that two expressions are equal. It has an equal sign. Examples: $40 + 25 = 65$ $(16 \div 4) - 3 = 1$

We simplify an expression or equation by performing operations to combine like terms.

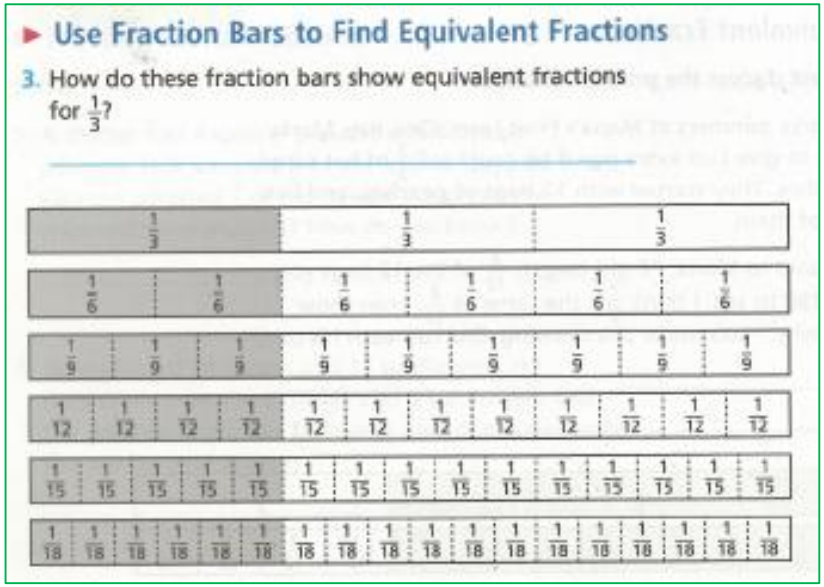
Use the Identity Property to simplify each expression.

1. $n + 5n =$ _____
2. $17t + t =$ _____
3. $x + 245x =$ _____
4. $9e - e =$ _____
5. $8c + c + c =$ _____
6. $(5z - z) - z =$ _____

Solve.

7. $30 \div (35 \div 7) =$ _____
8. $(72 \div 9) \div 4 =$ _____
9. $80 \div (32 \div 8) =$ _____
10. $13 - (9 - 1) =$ _____

4.NF.A Extend understanding of fraction equivalence and ordering. (Grade 4 expectations limited to denominators 2, 3, 4, 5, 6, 8. 120, 12, and 100



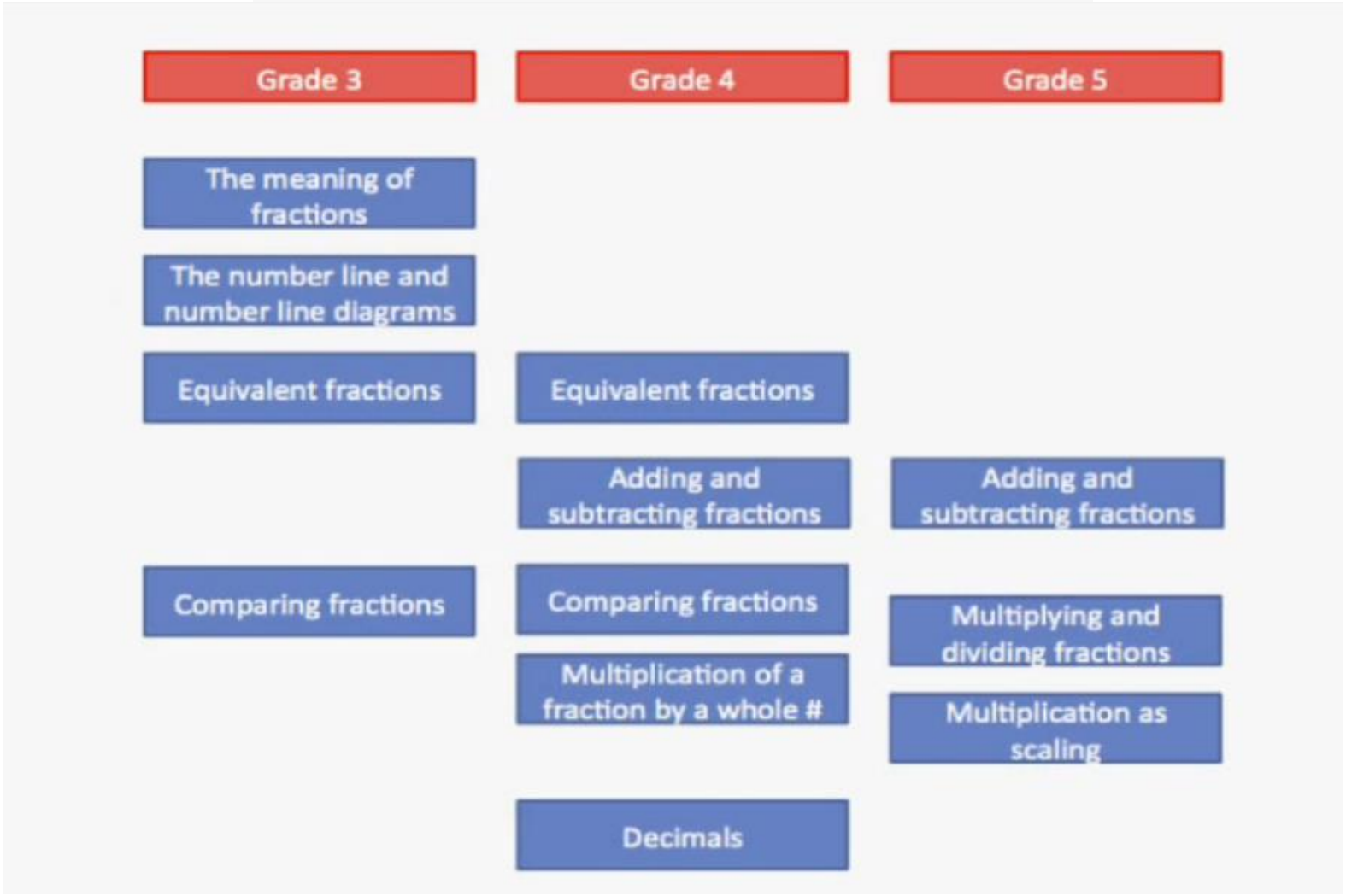
► Use Fraction Bars to Find Equivalent Fractions

3. How do these fraction bars show equivalent fractions for $\frac{1}{3}$?

Coherence Across the Grades

K.OA.4: **For any number from 1 to 9, find the number that makes 10** when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	1.OA.6: **Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.** Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	2.OA.2: **Fluently add and subtract within 20 using mental strategies.** (Note: See standard 1.OA.6 for a list of mental strategies). By end of Grade 2, know from memory all sums of two one-digit numbers.

Fractions Progression Module



A Closer Look into the Domains….

3.NF.3: d. **Compare two fractions with the same numerator or the same denominator** by reasoning about their size.	4.NF.2: **Compare two fractions with different numerators and different denominators,** e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.	5.NF.1: **Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions** in such a way as to produce an equivalent sum or difference of fractions with like denominators.

Progression of Fraction Multiplication in CCSS

Grade 4

4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Grade 5

5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

Grade 6

6.NS. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Activity: Fraction Card Sort

- Pause and access Coherence Activity 1.
- Without referencing the standards, sort the fraction cards based on the grade level in which the content appears in the standards.
- While completing the activity think about the following:
  - What was challenging about sequencing the tasks?
  - What information did you use to determine the grade-level placement of a task?
  - How does this activity add to your understanding of Coherence in the Standards?

Coherence in the Middle Grades

6.EE.7: **Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q*** for cases in which *p*, *q* and *x* are all nonnegative rational numbers.	7.EE.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. **Solve word problems leading to equations of the form *px* + *q* = *r* and *p*(*x* + *q*) = *r*, where *p*, *q*, and *r* are specific rational numbers.**	8.EE.8: Analyze and solve pairs of simultaneous linear equations. b. **Solve systems of two linear equations in two variables algebraically,** and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.

A Look Across the Conceptual Categories

Algebra 1	*Geometry*	*Algebra 2*
A.CED.1 **Create equations in one variable** and use them to solve problems.	G.GPE.1**: Derive the equation of a circle of given center and radius using the Pythagorean Theorem;** complete the square to find the center and radius of a circle given by an equation.	F.BF.1 **Write a function that describes a relationship between two quantities.**

What does student learning look like if the teacher is building coherence?

Look Fors

- Students experience a launch or initial review that anchors them before diving into new material.
- Students make comments like, “Oh! This is like when we…” or “It’s the same as when we used decimals” or “Last year we…”
- Students notice and ask about connections.
- Students can explain how some other concept they’ve studied is related to the current one.

Resources for Progressions

- http://ime.math.arizona.edu/progressions/
  - Updated versions of the early progressions drafts, revised and edited to correspond with the Standards
- http://ctcorestandards.org/?page_id=2
  - Compilation of the progression of standards by domain for K-2, 3-5 and 6-8
  - Compilation of the progression of standards by conceptual category for the traditional pathway approach to high school mathematics

Coherence Map

- Build student understanding by linking together concepts within and across grades.
- Identify gaps in a student's knowledge by tracing a standard back through its logical pre-requisites.
- Visualize and understand how supporting standards relate to the major work of the grade.

Coherence Take-Aways

Coherence allows for:

- Linking work to major focus areas of the grade level.
- A natural progression of content from grade to grade, course to course.
- Support of mathematical connections between and among domains

Thank You

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