The Shifts for CCS – Math

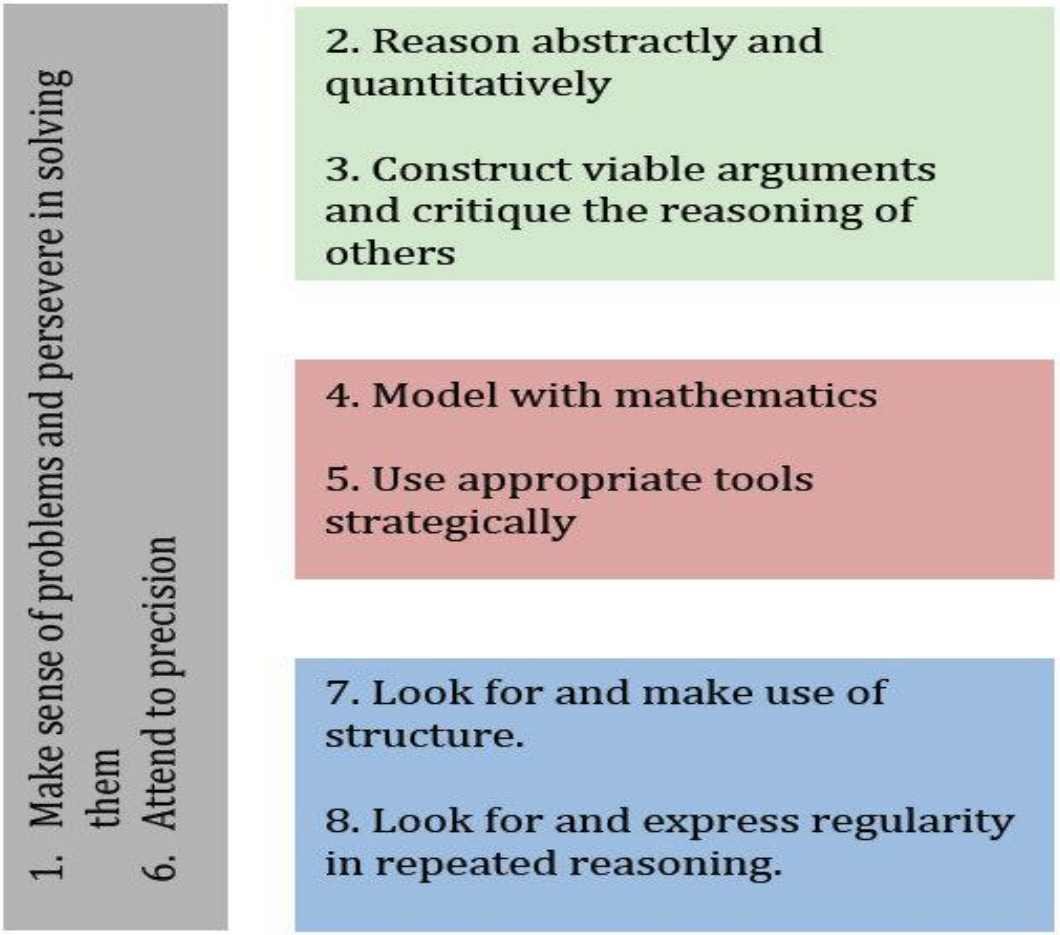
A Webinar Series

A Look Into The Standards

What's in the CCS-Math?

- The Standards for Mathematical Content
  - What students should know and be able to do
  - Content Domains
  - Domains into clusters
  - Clusters into standards
- The Standards for Mathematical Practice
  - The habits of mind

Organization of the Standards for Mathematical Practice



The *Standards for Mathematical Practice* were based on the NCTM Process Standards and the National Research Council's (NRC) Strands of Mathematical Proficiency.

Three Instructional Shifts for CCS - Mathematics

Focus on the Standards; teach less but for understanding.

Coherence – Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.

Rigor – Means a balance of solid conceptual understanding, procedural skill and fluency, and application of skills in problem solving situations.

FOCUS

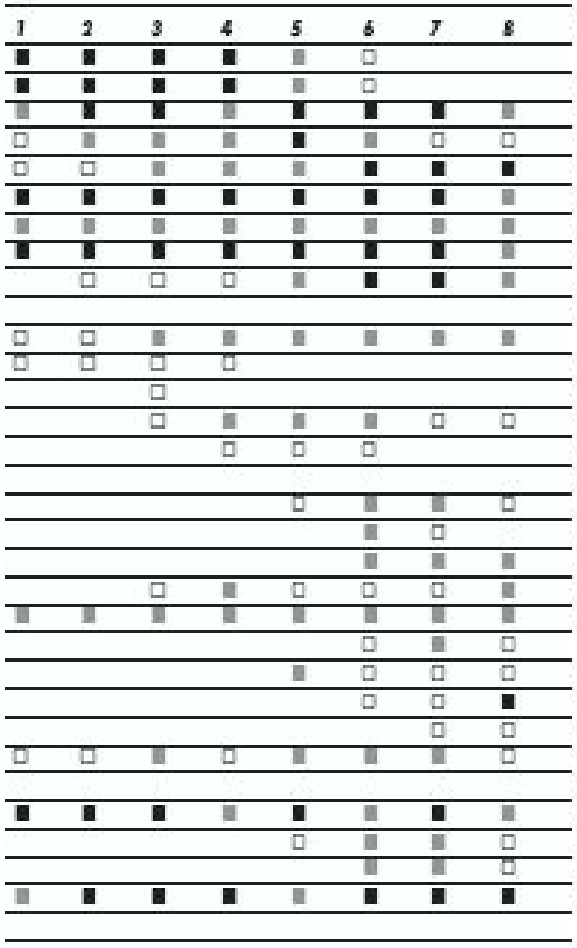
Shift 1: Focus

Focus strongly where the Standards focus

- ✓ Narrow the scope of content
- ✓ Focus deeply on what is emphasized in the Standards
- ✓ Move away from "mile wide, inch deep"
- ✓ Less topic coverage can be associated with higher scores on those topics covered

The Shape of Math in A+ Countries

Mathematics topics intended at each grade by at least two-thirds of A+ countries



Mathematics topics intended at each grade by at least two-thirds of 21 U.S. states

[1] *Schmidt, Houang, & Cogan, "A Coherent Curriculum: The Case of Mathematics." (2002).*

Focus Through Domains

Table 1. The Pre-CCSSM Strand Picture of Mathematical Content

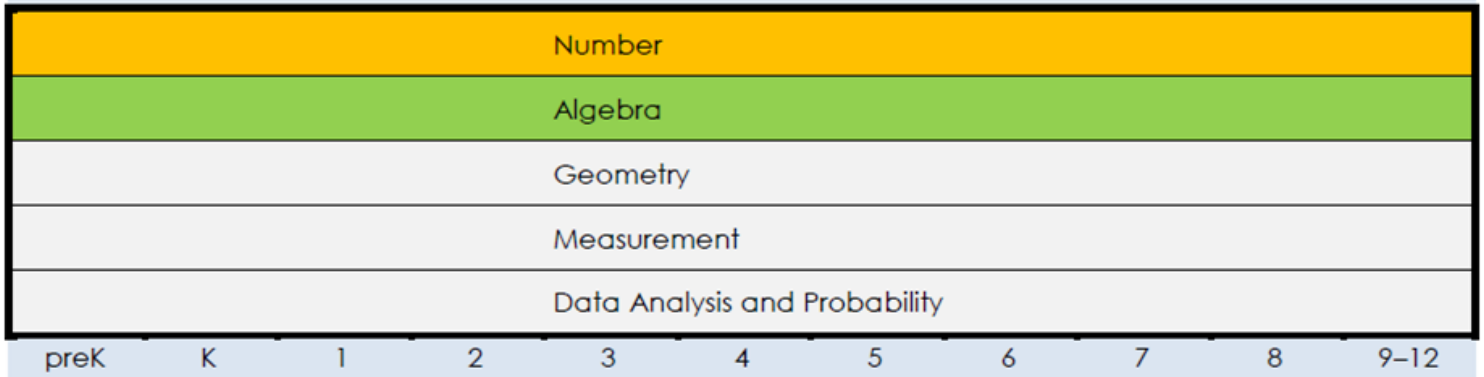
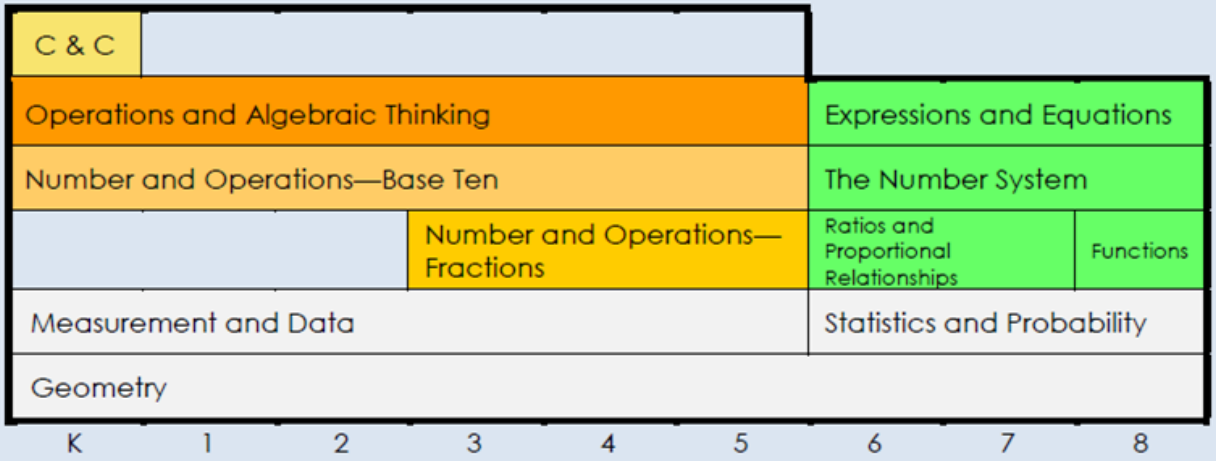


Table 2. The CCSSM Domain Picture of Mathematical Content



Levels of Focus

First level of focus: Knowing what is to be taught at each grade level and what is not.

- Serve as the foundation for the grade
- Essential mathematical ideas for each grade level
- Narrow the scope of content and deepen how time and energy is spent in the math classroom

Critical Areas

- 2 to 4 critical areas are identified at each grade from K-8
- 4 to 6 critical areas are identified at each course in a traditional pathway for high school
- Outline the essential mathematical ideas for each grade level or course
- Form a firm foundation on which to build concepts and procedures in later years and ultimately make students college and career ready

	Critical Areas		
K	Compare numbers	Use tally marks	Understand meaning of addition and subtraction
1	Add and subtract within 20	Measure lengths indirectly and by iterating length units	Create and extend patterns and sequences
2	Work with equal groups of objects to gain foundations for multiplication	Understand place value	Identify line of symmetry in two dimensional figures
3	Multiply and divide within 100	Identify the measures of central tendency and distribution	Develop understanding of fractions as numbers
4	Examine transformations on the coordinate plane	Generalize place value understanding for multi-digit whole numbers	Extend understanding of fraction equivalence and ordering
5	Understand and calculate probability of single events	Understand the place value system	Apply and extend previous understandings of multiplication and division to multiply and divide fractions
6	Understand ratio concepts and use ratio reasoning to solve problems	Identify and utilize rules of divisibility	Apply and extend previous understandings of arithmetic to algebraic expressions
7	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	Use properties of operations to generate equivalent expressions	Generate the prime factorization of numbers to solve problems
8	Standard form of a linear equation	Define, evaluate, and compare functions	Understand and apply the Pythagorean Theorem
Alg.1	Quadratic inequalities	Linear and quadratic functions	Creating equations to model situations
Alg.2	Exponential and logarithmic functions	Polar coordinates	Using functions to model situations

	How Did You Do?		
K	Compare numbers	Use tally marks	Understand meaning of addition and subtraction
1	Add and subtract within 20	Measure lengths indirectly and by iterating length units	Create and extend patterns and sequences
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K-8 Standard Organization

The critical areas are identified for each grade and a description of the critical area is included.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

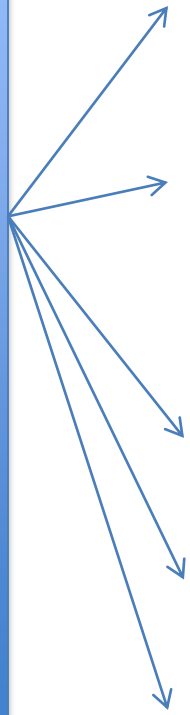
(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring

Traditional Pathway: High School Algebra I

Each course, outlined in the *Pathways* document is organized around *Critical Areas* (Units).



The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

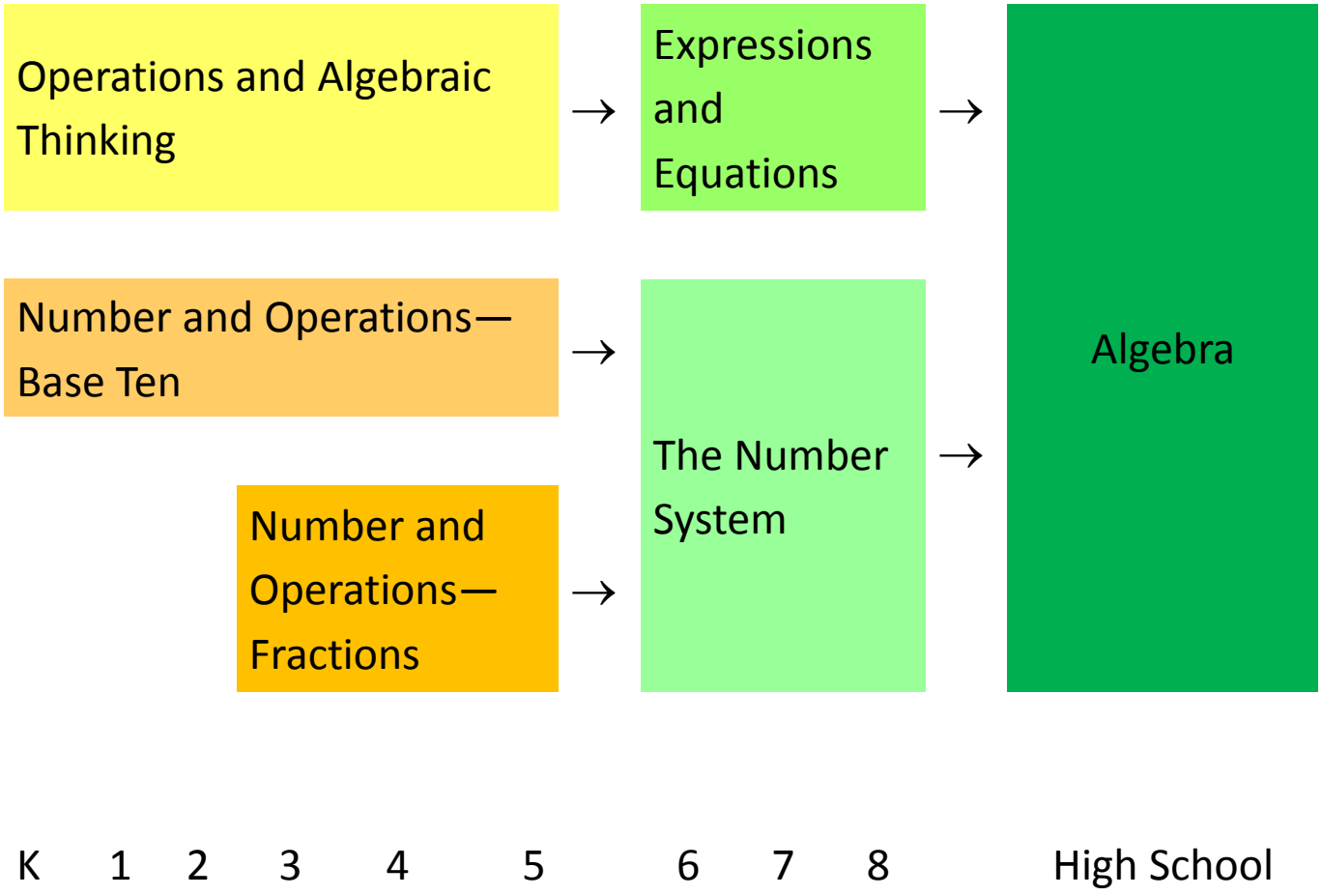
Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Examining the Critical Areas

- What are the identified critical areas for your grade or course?
- What are the important mathematical ideas for this critical area?
- Compare the concepts in the critical areas with those that you are currently teaching. In general, how are they similar? How are they different?
- How will you use the critical areas to guide your instruction?

Focus: Number and Operations



Crowding Out Focus

- **Probability**, including chance, likely outcomes, probability models *(introduced in grade 7)*
- **Statistical distributions**, including center, variation, clumping, outliers, mean, median, mode, range, quartiles; and statistical association or trends, including two-way tables, bivariate measurement data, scatter plots, trend line, line of best fit, correlation.*(introduced in grade 6)*
- **Coordinate transformations or formal definition of congruence or similarity** *(introduced in grade 8)*
- **Symmetry** of shapes, including line/reflection symmetry, rotational symmetry *(introduced in grade 4)*

Levels of Focus

Second level of focus: Knowing the major work of each grade.

- Not all content is emphasized equally
- Directly related to the critical areas
- Majority of the time should be dedicated to the major work of the grade

Practice with Grade 3

Look at the critical areas for each grade level and determine the specific skills and understandings found in each critical area.

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

Practice with Grade 3

Look for domains and clusters in which the skill or understanding is most likely to be found.

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Determining Major Work

Determine which standards in the cluster match the skill or understanding.

Represent and solve problems involving multiplication and division.
CCSS.MATH.CONTENT.3.OA.A.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*
Understand properties of multiplication and the relationship between multiplication and division.
CCSS.MATH.CONTENT.3.OA.B.5
Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
CCSS.MATH.CONTENT.3.OA.C.7
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Making it Simpler

Student Achievement Partners (SAP)

- Non-profit founded by three of the contributing authors of CCSSM
- Develops and makes available tools and resources free of charge

Cluster Emphases

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 4

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters ▢ Supporting Clusters ○ Additional Clusters

4.OA.A	■	Use the four operations with whole numbers to solve problems.
4.OA.B	▢	Gain familiarity with factors and multiples.
4.OA.C	○	Generate and analyze patterns.
4.NBT.A	■	Generalize place value understanding for multi-digit whole numbers.
4.NBT.B	■	Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NF.A	■	Extend understanding of fraction equivalence and ordering.
4.NF.B	■	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.C	■	Understand decimal notation for fractions, and compare decimal fractions.
4.MD.A	▢	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.B	▢	Represent and interpret data.
4.MD.C	○	Geometric measurement: understand concepts of angle and measure angles.
4.G.A	○	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Widely Applicable Prerequisites

Number and Quantity	Algebra	Functions	Geometry	Statistics and Probability	Applying Key Takeaways from Grades 6–8**
N-RN, Real Numbers: Both clusters in this domain contain widely applicable prerequisites. **N-Q★, Quantities**: Every standard in this domain is a widely applicable prerequisite. Note, this domain is especially important in the high school content standards overall as a widely applicable prerequisite.	**Every domain in this category** contains widely applicable prerequisites.° Note, the **A-SSE** domain is especially important in the high school content standards overall as a widely applicable prerequisite.	**F-IF, Interpreting Functions**: Every cluster in this domain contains widely applicable prerequisites.° Additionally, standards **F-BF.1** and **F-LE.1** are relatively important within this category as widely applicable prerequisites.	**The following standards and clusters** are relatively important within this category as widely applicable prerequisites: **G-CO.1** **G-CO.9** **G-CO.10** **G-SRT.B** **G-SRT.C** Note, the above standards in turn have learning prerequisites within the Geometry category, including: G-CO.A G-CO.B G-SRT.A	**The following standards** are relatively important within this category as widely applicable prerequisites: **S-ID.2** **S-ID.7** **S-IC.1** Note, the above standards in turn have learning prerequisites within 6-8.SP.	**Solving problems at a level of sophistication appropriate to high school by**: • Applying ratios and proportional relationships. • Applying percentages and unit conversions, e.g., in the context of complicated measurement problems involving quantities with derived or compound units (such as mg/mL, kg/m^3, acre-feet, etc.). • Applying basic function concepts, e.g., by interpreting the features of a graph in the context of an applied problem. • Applying concepts and skills of geometric measurement e.g., when analyzing a diagram or schematic. • Applying concepts and skills of basic statistics and probability (see 6-8.SP). • Performing rational number arithmetic fluently.

Table excerpted from the High School Publishers Criteria for the Common Core State Standards for Mathematics

What is a “large majority of time”?

At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the Major Work of the grade.

- K-8 Publishers’ Criteria, Spring 2013, p. 8

Coherence: Link to Major Topics Within Grades

Example: Data Representation

3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Supporting Work Related to Major Work

CCSS.MATH.CONTENT.4.MD.A.2

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money…

CCSS.MATH.CONTENT.4.OA.A.1

Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Elena has a cat with a mass of 4 kilograms. Ginger's cat has a mass that is 2 times as much as Elena's cat. What is the mass of Ginger's cat in grams?

Check for Understanding

- How would you summarize the major work of your grade or course?
- What would you have expected to be a part of the major work for your grade or course that is not?
- How can you would change your approach to teaching something designated as supporting the major work, instead of teaching it as a discrete topic?

Key Areas of Focus in Mathematics

Grade	Focus Areas
K–2	Addition and subtraction - concepts, skills, and problem solving and place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra and linear functions

Key Areas of Focus in Mathematics

Conceptual Category	Focus Clusters
Numbers and Quantity	Real numbers and Quantities
Algebra	Seeing Structure in Expressions, Arithmetic with Polynomial and Rational Expressions, Creating Equations, Reasoning with Equations and Inequalities
Functions	Interpreting Functions, Building Functions, and Linear, Quadratic and Exponential Models
Geometry	Similarity, Right Triangles and Trigonometry
Statistics and Probability	Interpreting Categorical and Quantitative Data and Making Inferences and Drawing Conclusions

Thank You

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