**Unit 8: Investigation 6 (3 Days)**

**The Golden Ratio**

**Common Core State Standards (extended)**

* G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)..
* G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Overview**

The Golden Ratio ($ϕ$) has been a source of fascination for millennia. Many consider rectangles with side lengths in the Golden Ratio the most aesthetically pleasing of all rectangles. The Golden Ratio also appears in two special isosceles triangles, pentagrams, and spirals. The study of the Golden Ratio introduces students to another mathematical constant that they can physically construct and algebraically manipulate.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Construct a rectangle with sides in the golden ratio.
* Use the divine proportion to find a value for the golden ratio.
* Explain algebraic relationships among different forms using the ratio.
* Create 72-72-36 and 72-54-54 golden triangles.
* Find repeated instances of golden triangles in pentagrams and related designs.
* Demonstrate that the ratio of Fibonacci numbers appears to converge to the golden ratio.

**Assessment Strategies: How Will They Show What They Know?**

* **Journal Entry 1** asks students to explain a construction of the Golden rectangle.
* **Journal Entry 2** asks students to explain the relationship between the golden ratio and the dimensions of a golden rectangle.

**Launch Notes**

Begin by showing students a variety of rectangles they could choose for a logo for their team. Ask them which rectangles seem more attractive and why. As on the opening page of Activity 8.6.1 include at least one square, at least one “elongated” rectangle (with a length : width ratio of at least 4 : 1) and one that is close to golden (length : width ratio ≈ 1.62)

**Teaching Strategies**

**Activity 8.6.1 Constructing a Golden Rectangle** engages students in using constructions (segment bisection, construct a perpendicular, copy a segment) to create a golden rectangle. Students then derive an algebraic expression for the ratio (phi φ) and explore some algebraic relations that φhas to itself.

**Differentiated Instruction (For Learners Needing More Help)** Calculating with radical expressions will be challenging for many students. Where appropriate, encourage students to use the approximate value $ϕ$ ≈1.618034 to verify identities. Be sure, however, that they understand that 1.618034 is not an exact value for $ϕ.$ For these students the exercises using Binet’s formula to find terms in the Fibonacci sequence (Activity 8.6.3) are probably not appropriate.

You may assign one or both **Journal Entries** following **Activity 8.6.1.**

In **Activity 8.6.2 Golden Triangles,** students investigate regular pentagrams. They find the golden triangle and other instances of phi. They prove some of these relationships.

**Activity 8.6.3 Fibonacci Sequences and the Golden Ratio** introduces real world situations that produce Fibonacci sequences. They study several features of these sequences including using a Fibonacci sequence to create a golden (logarithmic) spiral and the convergence of successive terms to phi.

In **Activity 8.6.4 A Spiral from Golden Rectangles** a way to construct a logarithmic spiral using the repeated construction of nested golden rectangles is introduced. Students learn to differentiate an Archimedean spiral from an equiangular (golden) spiral. [Huntley p. 101.]

Other avenues for exploration for Golden Ratios are found in **Activity 8.6.5 Extensions for the Golden Ratio**. These include more patterns with phi, continued fraction expressions for phi [Dalton p. 174-6], folding a regular pentagon, golden triangles in Penrose tiles, proving an alternative way to construct a golden section, and Fibonacci connections to nature. [Boles & Newman p. 60-61.] Students also create larger and smaller golden sections using similarity.

**Differentiated Instruction (Enrichment)** Activity 8.6.5 contains suggestions for independent research. Students may also want to consult the resources listed below.

**Journal Entry 1:** How do you use a square to construct a golden rectangle? Look for use of the Pythagorean Theorem to explain where $\frac{\sqrt{5}}{2}$ comes from.

**Journal Entry 2:** Use the words ‘length’ and ‘width’ to show the proportion from which the golden ratio is derived. Look for a proportion like $\frac{length+width}{length}=\frac{length}{width}$.

**Vocabulary**

Archimedean spiral

continued fraction

divine proportion

equiangular spiral

explicit formula

golden gnomon

golden mean

golden rectangle

Golden Ratio

golden section

golden triangle

phi, φ

recursive formula

**Resources and Materials**

Activity 8.6.1 Constructing a Golden Rectangle
Activity 8.6.2 Golden Triangles
Activity 8.6.3 Fibonacci Sequences and the Golden Ratio
Activity 8.6.4 A Spiral from the Golden Ratio
Activity 8.6.5 Extensions for the Golden Ratio

Dalton, LeRoy C. *Algebra in the Real World: 38 Enrichment Lessons for Algebra 2*. Palo Alto, CA: Dale Seymour Publications, 1983.

Hargittai, Istvan & Magdolna Hargittai. (1994). *Symmetry: A Unifying Concept.* Bolinas, CA: Shelter Publications, Inc.

Hilton, Peter & Jean Pedersen. (1988). *Build Your Own Polyhedra*. Menlo Park, CA: Addison-Wesley Pub’g Co. [How to fold a regular pentagon from long strip, pp. 18-21]

Huntley, H. E. (1970). *The Divine Proportion*. New York: Dover Publications, Inc.

Jacobs, Harold R. (1982). M*athematics: A Human Endeavor. Second Edition.* San Francisco: W. H. Freeman and Co.

Kappraff, Jay. (1991). *Connections: The Geometric Bridge Between Art and Science*. New York: McGraw Hill. (pp. 422 – 425)

Kinsey, L. Christine and Theresa E. Moore. (2002).  *Symmetry, Shape, and Space: An Introduction to Mathematics through Geometry.* Emeryville, CA: Key College Publishing.

Newman, Rochelle & Martha Boles (1987). *Universal Patterns.* Bradford, MA: Pythagorean Press.

Golden Ratio in nature:

<http://io9.com/5985588/15-uncanny-examples-of-the-golden-ratio-in-nature>

<https://originalbeauty.wordpress.com/2009/06/27/spirals-in-nature/>

Golden ratio in animated construction with spiral

<https://www.youtube.com/watch?v=zpaBD6Z6TOU>

Animation of spiral from nested golden triangles:

<http://britton.disted.camosun.bc.ca/geometry/whirling_triangles.html>