**Unit 8**

**Extensions and Projects**

**UNIT PLAN**

The Common Core State Standards for high school mathematics note that the itemized standards should constitute 85% of a curriculum. This unit builds upon the standards already addressed by probing deeper into various aspects of geometry.

The content of this unit consists of a variety of enrichment topics. Teachers may pick several of these topics to present to the entire class. These may done at any time during the school year or following Unit 7. The topics may presented in any order as they are all independent of each other. An alternative is to make the activities and the resources available to individual students or small groups for special projects.

The topics are organized as Investigations following the pattern of other units in the curriculum. The list below is not meant to be exhaustive. Other topics may be added at a later date.

Investigation 1: Frieze Patterns (builds on Unit 1)

Investigation 2: *Flatland* and the Fourth Dimension (builds on Unit 6)

Investigation 3: Further Investigation of Tessellations (builds on Unit 3)

Investigation 4: The History of Pi (builds on Unit 5)

Investigation 5: Fractals (builds on Unit 4)

Investigation 6: The Golden Ratio (builds on Unit 4)

Investigation 7: Semi-regular and Stellated Polyhedra (builds on Unit 6)

Most Investigation Overviews include a list of resources for students who wish to delve deeper into the topic.

**Overview of Investigations**

**Investigation 1 Frieze Patterns** builds on students’ understanding of transformations and the performance task from Unit 1. Students learn how to identify the seven types of frieze patterns and several ways to describe them, including Conway’s dance steps and the notation adopted by the International Union of Crystallography (IUC)

In **Investigation 2 *Flatland* and the Fourth Dimension** students begin with Edwin Abbott’s classic book on life in a two dimension world. Building on concepts introduced there, students explore properties of geometric figures in four dimensions and beyond.

Tessellations were introduced in Unit 3 Investigation 7. In this unit **Investigation 3 Further Investigation of Tessellations** has students systematically investigate ways that regular polygons may be arranged around a vertex and concludes by identifying the eight semi-regular tessellations. Extensions include Escher drawings and Penrose tiles.

In **Investigation 4 The History of Pi** students are introduced to various estimates of pi from ancient times, culminating in the work of Archimedes, who inscribed and circumscribed regular polygons, doubling the number of sides to arrive at his conclusion that 3$\frac{10}{71}$ < π < 3$\frac{1}{7}$.

In **Investigation 5 Fractals** students construct the Sierpinski Triangle, the Koch Snowflake and other self-similar figures.

**Investigation 6 The Golden Ratio** introduces students to the Golden Rectangle, the Golden Triangle, the Logarithmic Spiral, and Fibonacci numbers.

In **Investigation 7 Semi-regular and Stellated Polyhedra,** building on the study of regular polyhedra in Unit 6,students discover and classify the 13 Archimedean Solids and explore stellated polyhedra.

**Student Projects**

A course in Geometry provides a special opportunity for students to do mathematics projects that can easily integrate and extend their interests in visual arts, dance, crafts, computer graphics, architecture, music, fashion, and more. We hope that students are able to do at least one project in this course. We encourage students to consider this work something they would be proud to have presented publicly or online and as part of their learning portfolio.

Such projects have numerous benefits:

* Students have an opportunity to make choices and fashion work that shows off their learning.
* Students have an opportunity to work independently and/or collaboratively.
* Students have an opportunity to express themselves in a variety of ways: reading, writing, public speaking, hand or electronic graphics, construction, symbols, charts, etc.
* Teachers have an opportunity to ‘see’ students in a different way than regular classwork, homework, exit slips, and tests and to recognize and appreciate student achievement in an alternative mode.

Two of the authors found that every year when we did geometry projects there were students who had not previously ‘shone’ in geometry but who really dug in, found their voices, and excelled with a project. We hope that this is true, too, for you and your students.

The activities in Unit 8 build from and extend concepts, procedures, and ways of reasoning that have been growing across the course. We have created sequences of activities that set the stage for an array of related choices. In each Investigation the earlier activities lay the groundwork for the choices of extensions (projects) in the last activity. We have tried to identify valuable books and web links that provide more information and tools. Our hope is that the preliminary activities and the resources spur students to looking further at the topics.

We know that in today’s world with so much available on the web it is tempting for students not to do the reasoning and investigation for themselves that can develop the mathematical practices we value. In assigning projects teachers should ask students to explain their reasoning and where possible to ‘pose and investigate a related problem.’ Our experience in requiring this problem posing by students has given us deep insights, not only into their mathematical learning, but also into students’ confidence, insights, persistence, and other less-often-identified attributes of learning.

The file **Project Guidelines** contains sample student planning guides, student self-evaluation guides, a general project evaluation rubric, tips for student speakers, and a notes on orchestrating a Geometry Fair. We hope these materials give you starting points for engaging students in investigations and presentations in which both you and they can take pride.