**Activity 6.1.8a Convert Between Degrees and Radians**

In previous activities, you sketched an arc of a given length and the subtended central angle. You noticed that one revolution = 360° = 2π radians. By taking fractions of a circle, you identified both the radian measure and the degree measure for many central angles in standard position and the length of the arc subtended by that angle. How do you determine radians and degree measures when the arc is not a convenient fraction of the circle? In this activity, you will learn an algebraic process for changing back and forth between radians and degrees.

First let’s review how to convert between other units of measure that you probably already know: using the idea of a “conversion factor” and “unit analysis”. Then we will apply this method of converting between units of measure to converting between degrees and radians.

1. **How many inches in a foot? \_\_\_\_ .**
2. **Write this fact as an equation: “ \_\_\_\_foot = \_\_ inches”**
3. **Divide both sides of this equation by “12 inches”:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**
4. **Simplify the right hand side, and write the equation: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

The key idea is that $\frac{1 foot}{12 inches}=1$ , and when we multiply some value by “1” we maintain the value. The other key idea is that you can use algebraic thinking to manipulate words such as “foot” and “inches” the way you would simplify algebraic expressions with x’s and y’s. We will call $\frac{1 foot}{12 inches}$ a “conversion factor”, and we will use this factor that is equal to “1” to convert from inches to feet.

**Example: (convert feet to inches)**

Question: How many inches are in 2.5 feet? We have feet and we want inches, therefore multiply by something with “feet” in the denominator to “cancel” out the feet we want to “get rid of” will work. We want “inches” so introduce the word “inches” in the numerator. Choose to multiply by a fraction that is equal to “1”, and that has feet in the denominator and inches in the numerator.

 $2.5 feet\left(\frac{12 inches}{1 foot}\right)= $2.5(12 inches) = 30 inches

**Example: (convert inches to feet)**

The average height for a woman is 64 inches. How many feet is 64 inches?

64 inches = …?.... feet. Thinking: we want to multiply by a factor that is equal to 1 with inches in the denominator so it will “cancel out” the inches in the numerator.

64 inches ($\frac{1 foot}{12 inches})=\frac{64}{12} foot=5\frac{4}{12}feet=5\frac{1}{3}feet$

1. What was the conversion factor when you converted feet to inches? \_\_\_\_\_\_

f) What was the conversion factor when you converted inches to feet? \_\_\_\_\_\_

1. An iced tea container holds 12 cups. The recipe calls for 1 lemon for each quart of tea. How many lemons do I need? Fact: 1 quart = 4” cups

a. Rewrite cups as quarts. Show how you used the “conversion factor” method.

b. What is the conversion factor when changing cups to quarts? \_\_\_\_\_\_

c. What the conversion factor when changing quarts to cup?\_\_\_\_\_\_\_\_

2. The Farmer’s Cow ice cream store sells cups of ice cream from a huge bucket of ice cream. How many cups will be served from the 2.25 gallon bucket? Show that you used the conversion factor method to answer the question. (There are 16 cups in a gallon.)

**CONVERTING DEGREES TO RADIANS and RADIANS TO DEGREES**

3. Fact: 2π radians =360 degrees

a. Find a conversion factor for changing degrees to radians.

b. Change 90 degrees to radian measure using the conversion factor.

c. What is the conversion factor for changing radians to degrees?

d. Change $\frac{5π}{3}$ radians to degrees using the conversion factor.

e. At 1 o’clock, what angle is formed by the large hand and the small hand of a clock?

* in degrees? \_\_\_\_\_\_
* in radians\_\_\_\_\_\_\_

f. To convert radians to degrees, multiply by: \_\_\_\_\_\_\_\_

g. To convert degrees to radians, multiply by:\_\_\_\_\_\_\_\_

4. Do the following conversions. Fill in the blank. Then sketch the designated angle in standard position on a coordinate plane. Use simplified fractions, not decimals.

a. $\frac{7π}{6} radians= \\_\\_\\_\\_\\_\\_\\_degr$ees b. $-\frac{9π}{4} radians= \\_\\_\\_\\_\\_\\_\\_degr$ees

c. 135° =\_\_\_\_\_\_ radians d. -270° = \_\_\_\_\_\_\_\_\_ radians

d. 2π radians = \_\_\_\_\_\_\_\_\_° e. 0° = \_\_\_\_\_\_\_\_\_\_\_radians

When describing the measurement of something (length, volume, temperature, angle measure,…), it is customary to write the unit of measure EXCEPT for radians. An angle measure of $\frac{5π}{3}$ is understood to be in radians, whereas an angle of $\frac{5π}{3}$ ° measured in degrees because it is so marked. Feel free to write in the word “radians” or an abbreviation like “rad” when working with radians, if it helps.

5. Convert between radians and degrees. You may use decimal approximations when doing the following conversions, because they are not the special angles formed by dividing a circle in eights or twelfths. Remember that if there is no unit of measure, we mean “radian”. Round decimals to the nearest 100th .

1. 100$°$ ≈\_\_\_\_\_\_\_\_\_ radians
2. $\frac{3π}{5}= \\_\\_\\_\\_\\_\\_\\_°$
3. 314$° $≈\_\_\_\_\_\_\_\_\_ radians
4. $1°$ ≈ \_\_\_\_\_\_\_\_\_ radians
5. 1 ≈\_\_\_\_\_\_\_\_ degrees
6. 2 ≈\_\_\_\_\_\_\_\_degrees
7. 2π =\_\_\_\_\_\_ degrees

 6.

i. Do the following conversions. Fill in the blank. Use simplified fractions, not decimals.

ii. Sketch the designated angle in standard position on a coordinate plane.

iii. Write an angle between 0° and 360° and 0 and 2π that is co-terminal with the given angle. “Co-terminal” means the angles have the same terminal ray. You can add or subtract as many complete revolutions as needed to find a co-terminal angle.

iv. On the graph label the measure of the acute angle that is defined by the terminal ray of the angle and the x-axis.

**Example:**

 **i.** $\frac{15π}{4}$ **= \_675\_°**

$\frac{15π radians}{4} \left(\frac{180°}{π radians}\right)= \frac{15 }{2} \left(\frac{90°}{1}\right)=\frac{15 }{1} \left(\frac{45°}{1}\right)= 675°$

**Co-terminal angle in degrees:**

675 – 360= 315 (one rotation)

315° is an angle co-terminal with $675°$

Note: 315-360= -45, showing that 315° is 45° short of 360°

**ii Sketch**

675°

45°

**iii.** $\frac{15π}{4}$ **is co-terminal with angles** $\frac{7π}{4}$ **and 315°**

Co-terminal angle in radians:

Subtract $2π from \frac{15π}{4} by rewriting 2π as\frac{8π}{4}$:

$ $

$\frac{15π}{4}-\frac{8π}{4}=\frac{7π}{4} $

$\frac{7π}{4} $ is co-terminal with $\frac{15π}{4} $ . Note $\frac{7π}{4}-2π=-\frac{π}{4} $ showing that $\frac{7π}{4} $is $\frac{π}{4}$ short of 2π.

**iv. On the graph, label the measurement of the acute angle formed with x axis.** (the example shows the -45° angle indicated on the graph)

6a.

i. $ \frac{17π}{3}$ is coterminal with an angle of measure \_\_\_\_\_radians or \_\_\_\_\_ degrees.

ii Sketch angle in standard position.

iii. On the graph, label the measurement of the acute angle formed by the terminal ray of the angle and the x-axis.

6b.

i. $\frac{13π}{4}$ is co-terminal with an angle of measure \_\_\_\_\_\_\_ radians or \_\_\_\_\_\_\_\_degrees .

ii Sketch angle in standard position.

iii. On the graph, label the measurement of the acute angle formed by the terminal ray of the angle and the x-axis.

6c.

i. $-480°$ is co-terminal with an angle of measure \_\_\_\_\_\_\_\_\_ degrees or \_\_\_\_ radians

ii Sketch angle in standard position.

iii. On the graph, label the measurement of the acute angle formed by the terminal ray of the angle and the x-axis.

6d.

i. $-\frac{13π}{6} $is co-terminal with an angle of measure \_\_\_\_\_\_\_ radians or\_\_\_\_\_\_\_\_degrees .

ii Sketch angle in standard position.

iii. On the graph, label the measurement of the acute angle formed by the terminal ray of the angle and the x-axis.