**Unit 2: Investigation 2 (5 Days)**

**(Up to 9 additional days if Unit 8 was not completed Algebra 1)**

**Methods for Solving Quadratic Equations**

**Common Core State Standards**

A.SSE.3 Choose and produce an equivalent from of an expression to reveal and explain properties of the quantity represented by the expression\*

A.SSE.3a Factor a quadratic expression to reveal zeros of the function it defines.

A.SSE.3b Complete the square in a quadratic expression to revel the maximum/minimum value of the function it defines.

A.REI.4 Solve quadratic equations in one variable.

A.REI.4b Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions***.***

**Overview**

Investigation 2 builds on Unit 8 of the Algebra I curriculum and examines the various methods for solving quadratic equations; these include graphing, factoring, completing the square, and the quadratic formula. In this investigation, students will only explore rational and irrational solutions of quadratic equations. Students will conclude that a negative discriminant indicates no real solution. The absolute value function is defined as a piecewise function giving rise to the equivalence to an equation of the form . Absolute value functions and equations are also explored.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Solve quadratic equations using the following methods: graphing, square root method, factoring, completing the square, and the quadratic formula.
* Choose among the various methods for solving quadratic functions, and give reasons for their choices.
* Interpret the meaning of intercepts of a quadratic in the context of a real-world problem.
* Define the absolute value function as a piecewise defined function. Graph absolute value functions, use graphs to solve absolute value equations, and understand that.
* Determine when a quadratic equation has real number solutions or not.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 2.2.1** asks students to solve quadratic equations by factoring.
* **Exit Slip 2.2.2** asks students solve quadratic equations by completing the square.
* **Exit Slip 2.2.3** asks students to state the quadratic formula and use it to solve a quadratic equation
* **Journal Prompt 1** asks students to explain how to tell what the signs will be when factoring a trinomial (or difference of two squares that is a trinomial with 0 as the coefficient of the linear term).
* **Journal Prompt 2** asks students to explain how they know whether an equation is quadratic or linear, which method for solving the equation will work, and which method will work the best, depending on the equation.
* **Journal Prompt 3** asks students comment on a disagreement involving perspectives on the quadratic formula and completing the square method.
* **Activity 2.2.1** **Product of Two Lines** focuses on how the product of two linear functions is a quadratic function, and the two *x*-intercepts of the quadratic are the same as the *x*-intercepts of the two linear functions.
* **Activity 2.2.**2 **Multiplying and Factoring** is a refresher for students, they could do part for homework and part in their groups in class.
* **Activity 2.2.3** **Solve Equations by Factoring** provides opportunities to solve equations by factoring and to recognize another method is needed when factoring is not possible.
* **Activity 2.2.4** **Absolute Value,** will guide students through two definitions of the Absolute Value function (as distance from 0 on the number line and as a piecewise defined function) and students will identify the piecewise definition and the equation as being equivalent.
* **Activity 2.2.5** **Completing the Square** begins by having students solve equations of the form (*x* + c)2 = k by taking the square root of both sides of the equation and then solving for *x*.
* **Activity 2.2.6** **Deriving the Quadratic Formula** guides students to solve equations by completing the square and derive the quadratic formula.

**Launch Notes**

To motivate the desire to find the solution to a quadratic equation, ask four volunteers to come to the front of the room to illustrate four instances of projectile motion – all from the initial height of their hand stretched over their head: drop a crumpled paper ball, toss the ball straight up, toss the ball at a 75° angle to the floor, and toss the ball parallel to the floor.

Ask the class to: (a) sketch the graph of the trajectory of the projectile that is vertical position *y* as a function of horizontal position *x*; (b) sketch a graph of the height of the object as a function of time; (c) estimate the parameters for the function:

where *h* is the height of the projectile above ground, *t* is the time elapsed since the projectile was released, *g* is the force of gravity (use 9.8 m/s2, or 32 ft/s2), *vy*0 is the initial velocity in the vertical direction, and *h*0 is the initial height of the object; and (d) sketch a horizontal line to indicate various heights and write an equation to solve that determines when the projectile will be at a given height for either function.

If you choose, you can bring use a motion detector to find the equation for height as a function of time, and work with more accurate numbers for the parameters. You could also have students take a time-lapse video with a video camera to get a photo of the trajectory of the paper ball. They may have this feature on their tablet or cell phone. Please see the **Investigation 2.2 Teacher Launch Notes** and **Investigation 2.2 Student Launch Sheet** for more information.

**Overview**

Note About Unit 8 in the Algebra 1 Curriculum: Investigation 2 is both a review of solving quadratic equations from Unit 8 in the CT Common Core Algebra 1 curriculum and an extension of these concepts. This investigation goes beyond what is covered in Algebra 1 Unit 8, so do not skip this investigation. After doing **Activity 2.2.1 Product of Two Lines**, consider giving your students the **Pre-Test on Solving Quadratic Equations.** This pre-test assesses students’ ability to solve quadratic equations using the square root property, factoring, completing the square, and the quadratic formula. This will allow you to determine the depth of review that is needed.

If students only need a quick review of solving quadratic equations, then the materials in **Activities 2.2.2, 2.2.3, 2.2.5** and **2.2.6** should suffice. If students need more than the quick review of solving quadratic equations provided in the Unit 2 Investigation 2 activities, or if they did not complete Unit 8 in Algebra 1, then pull materials from Unit 8 Investigations 3, 4, 5, and 6 of the Algebra 1 Curriculum to review or teach solving quadratic equations. You can then assign parts of **Activities 2.2.2**, **2.2.3**, **2.2.5** and **2.2.6** as homework.

The following chart shows the Algebra 1 Unit 8 activities that correspond to Algebra 2 Unit 2 activities and the additional days that are needed to cover the Unit 8 activities.

|  |  |  |
| --- | --- | --- |
| **Algebra 2**  **Unit 2**  **Investigation 2**  **Activities** | **Preliminary**  **Algebra 1**  **Unit 8**  **Activities** | **Additional Days for Algebra 1**  **Unit 8**  **Activities** |
| Activity 2.2.2 Factoring  (1 day) | 8.5.1 Finding a Common Monomial Factor  8.5.2 Factoring Trinomials  8.5.3 Find Your Match | 3 days |
| Activity 2.2.3 Solve Quadratic Equations by Factoring  (Homework) | 8.5.4 Solving Quadratic Equations by Factoring | 1 day |
| Activity 2.2.5 Completing the Square  (1 day) | 8.3.2 The Square Root Property  8.3.3 Solving Two Step Equations with the Square Root Property  8.3.4 Solving Multistep Equations with the Square Root Property  8.3.5 Finding the *x*-intercepts of a quadratic function in vertex form  8.6.1 Completing the Square | 4 days |
| Activity 2.2.6 Quadratic Formula  (1 day) | 8.6.2 Proving the Quadratic Formula  8.6.3 Using the Quadratic Formula  If you taught a substantial portion of Unit 8, consider giving students the Unit 8 End-of-Unit assessment at this time. | 1 day |

Whether you need to do a thorough teaching of quadratic equations using Unit 8 in the Algebra 1 curriculum, or whether you only need to provide a quick review of solving quadratic equations using Unit 2 Investigation 2 activities, be sure to implement all the activities in this investigation.

**Teaching Strategies**

**Activity 2.2.1 Product of Two Lines** helps students understand a quadratic function as the product of two linear factors. This activity is based on the article “Parabolas: the Product of 2 Lines” in the December/January 2015 Mathematics Teacher. This activity focuses on how the product of two linear functions is a quadratic function, how the two *x*-intercepts of the quadratic are the same as the *x*-intercepts of the two linear functions, and sets the stage for solving equations by factoring, understanding end behavior of a quadratic function, and, eventually, the Fundamental Theorem of Algebra in Unit 3.

NCTM’s *Illuminations* website has similar lessons, including higher degree polynomials, called “Building Connections” at <http://illuminations.nctm.org/Lesson.aspx?id=1091> under Lessons, algebra, grades 9-12.

Before you distribute the student worksheet for **Activity 2.2.2 Multiplying and Factoring,** conduct a whole class discussion and demonstration about area models for multiplying and factoring. For the area model, refer to Unit 8 Investigation 4 “Quadratic Functions in Factored Form” and Investigation 5 “Factoring Quadratic Trinomials in the Connecticut Core Algebra 1 Curriculum. Another source for multiplying binomials using an area model is the Monterey Institute: <http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U08_L2_T3_text_container.html>. For factoring trinomials see page 9.5 and following at: <http://www.montereyinstitute.org/courses/Algebra1/PD9_RESOURCE/Algebra%20I_PD_U09_InstrGuide_v1.1.pdf>

Then review one example of solving a quadratic equation by factoring one example each for solving a quadratic equation like done by factoring, completing the square, and using the quadratic formula. Also review solving an equation of the form 3*x*2=12. If your students have some familiarity with solving quadratic equations, give them the **Unit 2 Investigation 2 Pre-Test** at this point, perhaps leaving the quadratic formula on the board for the students to reference. If students have not covered the material in Algebra 1 Unit 8, go back to those materials as indicated in the chart at the beginning of this Unit 2 Overview.

This discussion and Pre-Test will help you assess how much preliminary work from Unit 8 in the Algebra 1 curriculum would be helpful before doing **Activity 2.2.2 Multiplying and Factoring**, **Activity 2.2.3 Solving Equations Factoring**, **Activity 2.2.5 Completing the Square** or **Activity 2.2.6 Quadratic Formula** in this Investigation.

If **Activity 2.2.2 Multiplying and Factoring** is a refresher for students, they could do part for homework and part in groups during class. **Activity 2.2.2 Multiplying and Factoring** reviews multiplication of a binomial by a monomial using the distributive property. Then multiplying binomials is seen as the distributive property extended to two monomials times a binomial. Finally, factoring is seen as the inverse operation of multiplying.

**Journal Prompt 1** (Assign after **Activity 2.2.2**).

Ask students to explain how to tell what the signs will be when factoring a trinomial (or difference of two squares that is a trinomial with 0 as the coefficient of the linear term).

They might want to use the following problems to illustrate their explanation:

Multiply (x+5)(x+7); (x-5)(x+7); (x+5)(x-7); (x-5)(x-7);

Factor x2+12x+35; x2-12x+35; x2+2x-35; x2-2x-35, x2-25

Student responses should address that when the constant term is positive it could come from one of two situations and the middle term’s sign tells you whether two positive or two negative numbers are needed. A negative constant term tells you the signs are different and then they need to find a way to tell you how the sign of the middle term can help.

Provide a motivation for **Activity 2.2.3 Solve Equations by Factoring** by using an example similar to the situation in the launch -projectile motion, height as a function of time. In order to get an easily factorable equation we use an example in terms of feet and seconds. Given that the standard acceleration due to gravity is 32 ft/sec2, have students explain what is happening to the projectile in this function: h(t) = -16t 2+48t. (Ans: -16 is half the standard gravity, projectile is launched from the ground with an initial velocity in the vertical direction of 48 ft/sec.) Sketch a graph of the function and ask the students at what time the projectile is at height 10 ft? 32 ft? 36 ft? 0 ft? For each height, draw the horizontal line through the parabola and write the equation with 32, then 36, then 0 substituted in for height. Ask the students what is it that we are finding (in the context of the projectile) Solve for h=32 and for h=0 by factoring. Note that the solutions to a quadratic equation that is set equal to 0 are the *x*-coordinates of the *x*-intercepts of the graph of the function, if any. Can we solve -16t 2+48t =10? For now, we have to satisfy ourselves with estimating some solutions by graphing. Tell students that we will have to develop a method to use when factoring does not work.

You are showing a graphical interpretation for solving quadratic equations with zero and a non-zero constant on one side i.e. the solution of a quadratic equation is the intersection of the quadratic function with the horizontal line *y* = c. Note that if *y* = 0, then we are finding the *x*-intercepts of the graph of the function and the zeros of the function. Note that solving equations where one side of the equal sign is a constant is related to finding the *x*-coordinate of a related function where various horizontal lines intersect the graph–i.e., finding when the function takes on given values. Note that subtracting a non-zero constant from each side of the equation is analogous to a shifting the graph vertically so that the horizontal line moves to the *x*-axis.

Distribute **Activity 2.2.3 Solve Equations by Factoring**, having students work the problems for homework or in groups in class. The problems in **Activity 2.2.3 Solve Equations by Factoring** will give practice for setting one side of the equation equal to zero when solving by factoring. The factoring method works, because our number system has no zero divisors. The activity includes problems that are already in factored form with 0 on one side of the equal sign; some problems with 0 on one side and some not. Some problems will be factored and set equal to a non-zero constant. Students will also have a mixed practice section that includes linear equations (after simplifying) as well as factorable and non-factorable quadratic equations. The activity ends with fill-in-the blank questions that lead students to explain how to decide whether an equation is linear or quadratic, and how to solve each type of equation. Another type of question is working backward from the solution to the factored form of the equation. For example, if the solutions to a quadratic equation are 5 and 3, write a possible equation.

Assess student learning of solving quadratics by factoring with **Exit Slip 2.2.1**.

**Group Activity**

For the review and drill components of these activities, you may want to use group configurations to get student moving around and interacting with different students. A speed dating style set up may work, if you can divide some problems into 7 groups so that each group will take roughly 5 minutes or less of time per pair. One problem could have 2 parts: multiply (*x*+2)(*x*-5) and then factor *x*2-3*x*-10 and *x*2+3*x*-10 That will allow you to form 7 dating pairs in two groups, and will take 35 minutes.

**Differentiated Instruction (For Learners Needing More Help)**

Use a jigsaw puzzle style of group work by giving each person in a given heterogeneous group a card colored by ability (the ability coding is unbeknownst to the students). Organize temporary homogeneous groups by pulling together the students having the same color card. Assign a problem to each group by level of difficulty. Everyone should experience success and be done about the same time. Reconvene the original heterogeneous groups and have each student present their problem and its solution to the original group.

To pave the way for the method of “completing the square”, students should understand absolute value and why . Solving absolute value equations is a requirement of the Core Curriculum and fits in here. **Activity 2.2.4 Absolute Value,** , will guide students through two definitions of the Absolute Value function (as distance from 0 on the number line and as a piecewise defined function) and students will identify the piecewise definition and the equation as being equivalent. Some discussion should ensue about what is a definition and what is a theorem that follows from a definition. Note that any of the three statements about absolute value (distance from 0, piecewise function, square root of a number squared) could be chosen as the one definition of absolute value and the other two statements would follow from the definition. The activity will also note that both the absolute value function and the quadratic function are even, two-to-one functions, (not 1-1). In this activity, students will come to understand that the radical symbol means the principal root. Therefore, since there are two solutions to the equation *x*2 = c, for ‘c’ a non-zero constant, the +/- symbol needs to be put in front of the radical symbol. Be sure students understand that the +/- is a shortcut for writing two distinct solutions by requiring students to write the two solutions separately at times, not always as the compound statement with the +/- symbol.Teacher notes provide more detail.

Before you distribute the paper for **Activity 2.2.5 Completing the Square,** lead students through the lesson on NCTM Illuminations “Proof Without Words: Completing the Square” at <http://illuminations.nctm.org/Activity.aspx?id=4212>

Now distribute **Activity 2.2.5 Completing the Square** that begins by having students solve equations of the type (*x* + c)2 = k by taking the square root of both sides of the equation and then solving for c. Students see how nice it is to solve such equations – and any constant can be on one side of the equation. Next, students are asked to square binomials of the form (*x* + c)2 for c=1, 2, 3, 4… 8…15. By pattern recognition, students will then find the missing numbers in a sentence of the form provided 1 out of the 3 numbers is given: (*x* + ?)(*x* + ?) = *x*2 +\_?\_*x* + \_?\_ . Ask students to articulate how they filled in the blanks. Next, students learn and practice solving quadratic equations by completing the square for the simpler case of quadratics that have a coefficient of 1 for the quadratic term. Be sure to assign a few problems that lead to fractions in the constant term. Challenge the students to solve *x*2+ b*x*+ c = 0 by completing the square, using the letters b and c instead of numerical values.

At the end of **Activity 2.2.5 Completing the Square**, are some fill in the blank and true-false sentences for students to do. These sentences could serve as discussion questions to clarify and solidify the student learning.

Note: See the article <http://eric.ed.gov/?id=EJ890225> by Yaremah and Hendricks in AMATYC Journal MathAMATYC Educator Vol 1. No. 3 May 2010 – Using a Square to Complete the Algebra Student – Exploring Algebraic and Geometric Connections in the Quadratic Formula.

**Differentiated Instruction (For Learners Needing More Help)**

Some students may benefit from scaffolding some of the problems for each part of **Activities 2.2.2, 2.2.3**, and **2.2.4**. You may especially want to prepare some scaffolded exercises for completing the square involving fractions and completing the square for

*x*2 + b*x* + c = 0.

**Following Activity 2.2.5** use **Exit Slip 2.2.2** to assess student understanding of the completing the square method.

**Journal Prompt 2** (Assign after **Activity 2.2.5**)

Students have learned many methods for solving equations- both linear and quadratic equations. Ask students to explain: (a) how they know whether an equation is quadratic or linear; (b) which methods for solving will work (graphing, isolating the variable, completing the square); and (c) which method works best, depending on the equation. Students must justify their answers.

Students may want to illustrate their explanations by using equations similar to the following four equations: x2 +8x-4=0; x(x+8) - (4+x)=16 – x; x2+x–1 = 0;

3(x+5) = 2(x+10); 10x2 = 250.

Students may explain what methods worked, what did not work and why. Students can state what method they preferred and why.

As you distribute **Activity 2.2.6 Deriving the Quadratic Formula** tell students that they are going to discover a shortcut for completing the square. **Activity 2.2.6 Deriving the Quadratic Formula** guides students in solving an equation like 5x 2+ 6x+ -2= 0 by completing the square, and by substituting the parameters a, b and c in place of 5, 6 and 2 students derive the quadratic formula for themselves. **Activity 2.2.6a Deriving the Quadratic Formula** provides more scaffolding for students who may need more support. You may work the first part of the activity as whole group discussion or let the students work the entire activity in groups. Either way, ask questions to be sure that students see the parallel structure between the solutions to the specific quadratic equation and the general quadratic equation. Verify that the quadratic formula works by substituting a=5, b=6 and c=-2. Ask what would happen if there were a negative number in the radicand. (Answer: it is not a real number). Point out that if the number in the radicand is negative, we will say “no real solution”. In a future investigation, students will learn how to tell the nature of the roots according to the discriminant. Now you can distribute **Activity 2.2.6 Deriving the Quadratic Formula** which will guide students through problems using the quadratic formula. You may want to pick up where the launch left off by solving a few equations from the launch. Try h(t)=-4.9t2+0t+2 , for heights 2, 1 and 0 meters. Or

h(t) = -4.9 t2 + 4.5 t + 2 for heights 2, 3.03, 4 and 0 (note: 4 is no real solution).

You may also want to revisit the question that Galileo answered: If two balls with different weights are dropped from a height, will the heavier one reach the ground sooner than the lighter one? (Answer: no, see physics websites suggested in the resources at the end of this overview.)

Determine to what extent you need to re-teach the quadratic formula by giving **Exit Slip 2.2.3** which asks students to first state the quadratic formula, then for 3*x*2+*x*+15=20, identify a, b and c, and then solve the equation.

**Differentiated Instruction (Enrichment)**

Ask students what percentage of all possible quadratic equations (one side set equal to zero) can be solved by factoring. Have a group of students work on an exhaustive search for factorable quadratic equations. To get a handle on this question pursue a similar problem by limiting the coefficients to integers between +/- 9 and restricting the quadratic coefficient to a positive integer. Let the students think about this for a few days before you give hints. One approach would be to list all possible quadratic trinomials following some pattern such as listing all trinomials for a=1 and for b=1

1x 2+1x+1 does not factor (DNF) 1x 2+1x-1 DNF

1x 2+1x+2 DNF 1x 2+1x-2 = (x-1)(x+2)

1x 2+1x+3 DNF 1x 2+1x-3 DNF

1x 2+1x+4 DNF 1x 2+1x-4 DNF

1x 2+1x+5 DNF 1x 2+1x-5 DNF

1x 2+1x+6 DNF ETC …

Next, list all trinomials for a=2 and b= -1

1x 2-1x+1 DNF 1x 2-1x-1 DNF

1x 2-1x+2 DNF 1x 2-1x-2 = (x+1)(x-2)

1x 2-1x+3 DNF 1x 2-1x-3 DNF

1x 2-1x+4 DNF 1x 2-1x-4 DNF

1x 2-1x+5 DNF 1x 2-1x-5 DNF

ETC …

Next, list all trinomials for a=1 and b=+/-2, all the way to b=+/-9

Next list all trinomials for a=2, and b=+/-1 thru +/-9, ETC…

**Differentiated Instruction (Enrichment)**

Pose the question whether or not there is a formula analogous to the quadratic formula for polynomials of degree 3 or 4 or more…. Students can research the existence and/or the historical search for a cubic formula, a quartic formula or a quantic formula. Their research will lead them to stories of mathematicians guarding their secret methods for solving equations quickly. Contrast the secrecy of in the past with today’s culture of sharing mathematical discoveries. Students can search the article by Richard W. Feldmann about the Cardano – Tartaglia Dispute regarding the cubic equation. It is from NCTM Mathematics Teacher, Vol 54. No. 3 (March 1961), pp. 160-163. <http://www.jstor.org/stable/27956338?seq=1#page_scan_tab_contents>

The story of Galois is interesting, too. He died from wounds suffered in a duel at the age of 20. The night before the duel, worried that he wouldn’t survive to tell the world about his mathematical discoveries, so he stayed awake all night to write a proof of a theory that bears his name. Galois Theory proves that there is no formula for the quantic equation.

**Journal Prompt 3**

Have students comment on the following disagreement between the squabbling Bernoulli brothers:

*Jacob*: We learned a completely new method for solving quadratics today…. the quadratic formula.

*Johann*: That is not a new method; it is the same as completing the square.

*Jacob*: Well, I think the quadratic formula is the best way to solve an equation, and it is the only way I am ever going to use.

*Johann*: As I said, it is not any different from completing the square, and sometimes there are easier ways. I can give you examples where quadratic formula is not the easiest way.

Ask the students to explain why Johann is right when he says the quadratic formula is the same as completing the square; and the quadratic formula is the only method you ever need to know. Give an example for when the quadratic formula is the only way to solve a quadratic equation.

Then the students have to explain why Jacob is right about there being easier ways to solve a quadratic equation than the quadratic formula. Give examples when the quadratic formula is not the easiest way to solve a quadratic equation.

For those who enjoy math history, have them do a bit of research on the Bernoulli Brothers <http://www.storyofmathematics.com/18th_bernoulli.html> or <http://www.math.wichita.edu/history/men/bernoulli.html>

**Closure Notes**

Ask students what methods of solving quadratic equations always work and what methods sometimes work. What methods give exact solutions, and which give approximate answers? What is the difference, if any, between completing the square and using the quadratic formula? How are the number of solutions to a quadratic equation related to the graph of the corresponding quadratic function and the factored form of the quadratic? A class discussion of the **Journal Prompt 3** responses would be a nice way to conclude the investigation.

**Vocabulary**

Absolute Value

Binomial

Completing the square

Distributive property

Factor

Factor a quadratic over the integers

Monomial

Piecewise defined function

Quadratic equation

Quadratic formula

Quadratic function

Radicand

Root

Solution

Zeros of a function

**Resources and Materials**

**Activities 2.2.1, 2.2.2, 2.2.3, 2.2.4, 2.2.5 and 2.2.**6 should be completed in this investigation.

Activity 2.2.1 Product of Two Lines

Activity 2.2.2 Multiplying and Factoring

Activity 2.2.3 Solve Equations by Factoring

Activity 2.2.4 Absolute Value,

Activity 2.2.5 Completing the Square

Activity 2.2.6 Deriving the Quadratic Formula

Unit 2 Investigation 2 Pre-Test

Investigation 2.2 Teacher Launch Notes

Investigation 2.2 Student Launch Sheet

A nice animation for understanding that the independence of vertical motion and horizontal motion for projectiles motion is available at The Physics Classroom, Parabolic Motion of Projectiles <http://www.physicsclassroom.com/mmedia/vectors/bds.cfm> ,and the “The Monkey and the Zookeeper” at <http://www.physicsclassroom.com/mmedia/vectors/mzi.cfm>

If you want more background information about the horizontal and vertical components of projectile motion see <http://www.physicsclassroom.com/class/vectors>, [the Physics Classroom](http://www.physicsclassroom.com/) » [Physics Tutorial](http://www.physicsclassroom.com/class) » Vectors - Motion and Forces in Two Dimensions

“Three Lessons on Parabolas” by Ben Ceyanes, Pamela Lockwood, and Kristina Gill. NCTM Mathematics Teacher December 2014/January 2015 “ Vol 108, No 5, page 369

Geometry software such as Geogebra or Geometers Sketchpad

NCTM Illuminations “Proof Without Words: Completing the Square” at <http://illuminations.nctm.org/Activity.aspx?id=4212>

“The Cardano-Tartaglia Dispute”, by Richard W. Feldmann, Jr.,*The Mathematics Teacher* , Vol 54, No. 3, March 1961, pp. 160-163. Stable URL: <http://www.jstor.org/stable/27956338>

“Using a Square to Complete the Algebra Student: Exploring Algebraic and Geometric Connections in the Quadratic Formula”, Yarema, Connie H.; Hendricks, T. David, MathAMATYC Educator, v1 n3 p30-35 May 2010, <http://eric.ed.gov/?id=EJ890225>

<http://www.storyofmathematics.com/18th_bernoulli.html> or <http://www.math.wichita.edu/history/men/bernoulli.html>

<http://illuminations.nctm.org/Lesson.aspx?id=1091>

<http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U08_L2_T3_text_container.html>

For factoring trinomials see page 9.5 and following at: <http://www.montereyinstitute.org/courses/Algebra1/PD9_RESOURCE/Algebra%20I_PD_U09_InstrGuide_v1.1.pdf>