**Unit 6: Investigation 3 (3 Days)**

**Volume**

***CCSS:***

**G-GMD.1**. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.

**Overview**

Apply formulas for the areas of plane figures to compute volumes of prisms, pyramids, cylinders, and cones.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Understand Cavallieri’s principle and how it is applied to finding the volumes three-dimensional figures.
* Find the volumes of prisms, cylinders, pyramids, cones and frustums

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 6.3.1** asks students to apply Cavalieri’s principle and to find the volumes of prisms and cylinders.
* **Exit Slip 6.3.2** asks students to find the volumes of pyramids and cones.
* **Journal Entry** asks students to explain how decomposition is used to find volumes.

**Launch Notes**

The filling of transparent solid figures serves as the launch for this unit about volume.

Use a set of transparent hollow figures to demonstrate volume relationships.

A set like this is available from:
<http://www.hand2mind.com/item/clear-plastic-geometric-volume-set/2481>



Use colored water to fill the larger containers from the appropriate cone or pyramid. Ask students vote on how many pyramids full of water they think will be necessary to fill the cube with the same base and altitude. Similar explorations can occur with the cone and cylinder and the cone and sphere. Note that the hemisphere has the same volume as the cone whose height is twice the radius. Therefore the volume of the sphere is twice that of the cone.

**Teaching Strategies:**

**Activity 6.3.1** **Informal Explorations with Volume:** Students record their observations of the above demonstration and make conjectures about the formulas for volume. Later in this investigation they will study prisms, cylinders, pyramids, and cones more formally. The volume of a sphere is studied in more detail in Investigation 5.

In **Activity 6.3.2 Cavalieri’s Principle:** Students will explore shapes with congruent volume by using stacks of coins and layers of play dough. You will need to provide students with sets of 10 pennies, play dough, and dental floss. Students form shapes of play dough and make parallel cuts through the shapes to demonstrate the principle with cones, pyramids, and prisms. They conjecture that an oblique cylinder or cone has the same volume as a right cylinder or cone with the same base and the same height.A more thorough explanation can be found at [http://en.wikipedia.org/wiki/Cavalieri's\_principle](http://en.wikipedia.org/wiki/Cavalieri%27s_principle)

Patrick Honner is mathermatics teacher at Brooklyn Technical High School in Brooklyn, New York. developed a lot of interesting materials for his classes (see [http://mrhonner.com](http://mrhonner.com/)) He created the example of Cavalieri’s Principle using CD cases shown at the beginning of this activity.

In **Activity 6.3.3 Prisms and Cylinders** students began to use a more formal approach to finding volume. They start by postulating that the volume of a right rectangular prism is the product of the area of a base times the height. That postulate along with Cavalieri’s principle allows them to extend that relationship to triangular prisms and eventually to all prisms. Through an informal limit argument that relationship is extended to cylinders. In all cases the formula *V* = *Bh* applies.

**Differentiated Instruction (For Learners Needing More Help)**

Provide students who have difficulty visualizing three-dimensional object, concrete models to use as they work on various problems.

**Differentiated Instruction (Enrichment)** Ask students to use trigonometry to find the volume of a regular pentagonal pyramid inscribed in a cylinder with radius *r* and height *h*.

Following **Activity 6.3.3** you may give **Exit Slip 6.3.1.**

**Activity 6.3.4 The Volume Formula for Pyramids** starts with a proof that if two triangular pyramids have the same base and same height, then they have the same volume. This result is then used in a dissection proof of the **Pyramid-Prism Theorem:** the volume of a pyramid is one-third the volume of a prism with the same base and height. To aid in understanding the proof, students assemble three pyramids with the same volume that fit together to form a prism.

**Differentiated Instruction (For Learners Needing More Help)**

The proof in question 1 of Activity 6.3.4 is subtle. Some students may skip it if necessary, in which case you may postulate the Pyramid Volume Theorem.

In **Activity 6.3.5 Cones and Pyramids** students examine the relationship between a cone and an inscribed and circumscribed regular pyramid. As the number of sides of the base of the pyramid increases the volume of both pyramids approach that of the cone. An informal limit argument establishes that the formula *V* =$\frac{1}{3}$*Bh* applies to cones as well as to pyramids.

**Differentiated Instruction (Enrichment)** Ask students to show that the volume of a regular tetrahedron with edge = *e* is given by $e^{3}\frac{\sqrt{2}}{12}$.

Following **Activity 6.3.5** you may give **Exit Slip 6.3.2.**

If time permits, assign **Activity 6.3.6 Frustums of Cones and Pyramids.** Students will break shapes into component parts to find volumes and surface areas of frustums of cones, and pyramids. Students apply the concept of similarity, developed in Unit 4, to find dimensions of the smaller cone or pyramid which together with the frustum makes up the larger cone or pyramid.

**Journal Entry**

Explain how you use the decomposition of a solid to find the total volume. the volume of prisms and pyramids. Look for students to explain that pyramids with polygonal bases can be subdivided into triangular prisms and that a triangular pyramid may be subdivided into three pyramids.

**Closure Notes**

Have students make a flow chart showing how the volume formulas for prisms, pyramids, cylinders, and cones are all related to the Right Rectangular Prism Postulate.

**Group Activity**

To review this investigation, have students work in groups of 3. One person names a type of three-dimensional figure, the second person makes as sketch and gives the dimensions that are needed to find the volume, and the third person performs the calculations necessary to determine the volume. All group members check the work. Then rotate roles.

**Vocabulary**

circumscribed (prism or pyramid)
cone

cylinder

frustum (of cone or pyramid)

inscribed (prism or pyramid)

prism

pyramid

sphere

**Postulates and Theorems**

**Rectangular Prism Volume Postulate:** If *B* is the area of the base and *h* is the height of a right rectangular prism, then Volume = *Bh*

**Cavalieri’s Postulate:** If two solid figures lie between two parallel planes and every plane parallel to these two planes intersects both solids in cross-sections of equal area, then the two solid figures have equal volumes.

**Prism Volume Theorem:** The volume of any prism is found by multiplying the area of its base by its height.

**Pyramid Volume Theorem:** If the bases of two pyramids have the same area and if the heights of the pyramids are equal, then the pyramids have equal volumes.

**Pyramid-Prism Theorem:** The volume of a pyramid is one-third the volume of a prism with the same base and height.

**Resources and Materials**

Transparent shapes and materials to fill them (Activity 6.3.1)

Dental floss

Play dough

Pennies (Activity 6.3.2)

Template for Activity 6.3.4 (black and white and color versions)

6.3.1 Informal Explorations with Volume

6.3.2 Cavalieri’s Principle

6.3.3 Prisms and

6.3.4 (Volume Formula for Pyramids

6.3.5 Cones and Pyramids

6.3.6 (Frustums of Cones and Pyramid

Exit Slip 6.3.1

Exit Slip 6.3.2