**Activity 5.8.3 Deriving the Standard Equation of an Ellipse**



We will derive the equation of an ellipse in standard form. The foci are at (–*c*, 0) and (*c*, 0). The endpoints of the major axis are (–*a*, 0) and (*a*, 0). The endpoints of the minor axis are (0, –*b*) and (0, *b*).

1. Sum of the distances from the point to the foci is constant. Why?
2. Constant = $\sqrt{(x-(-c))^{2}+(y-0)^{2}}+\sqrt{(x-c)^{2}+(y-0)^{2}}$ using the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ formula.

If you move point *P* so that it coincides with point *E*, you can see that the sum of the distances is equal to the length of the major axis or 2*a*.

Therefore 2*a* = $\sqrt{(x-(-c))^{2}+(y-0)^{2}}+\sqrt{(x-c)^{2}+(y-0)^{2}}$

If you move point *P* so that it coincides with point *B*, you get a right triangle where a is the length of the hypotenuse and *b* and *c* are the lengths of the legs.

1. This gives us $a^{2}=\\_\\_\\_^{2}+\\_\\_\\_^{2}$

Using $2a=\sqrt{(x+c)^{2}+(y)^{2}}+\sqrt{(x-c)^{2}+(y)^{2}}$

 We get $2a-\sqrt{(x-c)^{2}+(y)^{2}}=\sqrt{(x+c)^{2}+(y)^{2}}$

$$4a^{2}-4a\sqrt{(x-c)^{2}+(y)^{2}}+(x-c)^{2}+y^{2}=(x+c)^{2}+(y)^{2}$$

 4. What did we do to get this last step?

Now expand terms on both sides of the equation and simplify:

$$4a^{2}-4a\sqrt{(x-c)^{2}+(y)^{2}}+x^{2}-2cx+c^{2}+y^{2}=x^{2}+2cx+c^{2}+(y)^{2}$$

$$4a^{2}-4a\sqrt{(x-c)^{2}+(y)^{2}}=4cx$$

Get the radical by itself on one side:

$$-4a\sqrt{(x-c)^{2}+(y)^{2}}=4cx-4a^{2}$$

$$-a\sqrt{(x-c)^{2}+(y)^{2}}=cx-a^{2}$$

 Square both sides again.

$$a^{2}((x-c)^{2}+(y)^{2})=(cx-a^{2})^{2}$$

 $a^{2}((x-c)^{2}+(y)^{2})=c^{2}x^{2}-2a^{2}c^{2}x^{2}+a^{4}$

$$ a^{2}x^{2}-2a^{2}c^{2}x^{2}+a^{2}c^{2}+a^{2}y^{2}=c^{2}x^{2}-2a^{2}c^{2}x^{2}+a^{4}$$

$$ a^{2}x^{2}+a^{2}c^{2}+a^{2}y^{2}=c^{2}x^{2}+a^{4}$$

$$ a^{2}x^{2}-c^{2}x^{2}+a^{2}y^{2}=a^{4}-a^{2}c^{2}$$

$$ x^{2}(a^{2}-c^{2})+a^{2}y^{2}=a^{2}(a^{2}-c^{2})$$

1. Now use$ a^{2}=b^{2}+c^{2}$, to make a simpler equation:
2. Show that this equation can be rearranged to give $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$

 $ x^{2}(b^{2})+a^{2}y^{2}=a^{2}b^{2}$

$$\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$$