**Activity 5.8.2 Constructing an Ellipse through Paper Folding**

Definition: An **ellipse** is the locus of points the sum of whose distances from two fixed points (called the **foci**) is a constant.

You’ll need the following materials for this activity:

1 piece of wax paper or parchment paper

Compass

Pen/Pencil

1. Follow these steps to carry out the construction.

1. On your piece of wax paper, use your compass to construct a fairly large circle. (Be sure

to make the radius small enough so that the entire circle is contained on the wax paper.

1. Label the center point of your circle. Label this point *F*1.
2. Plot and label another point in the interior of this circle. Label this point *F*2.
3. Plot approximately 20-25 points *on the circle*. (Just draw dots to represent these points).

Label *any one* of these points as *P*.

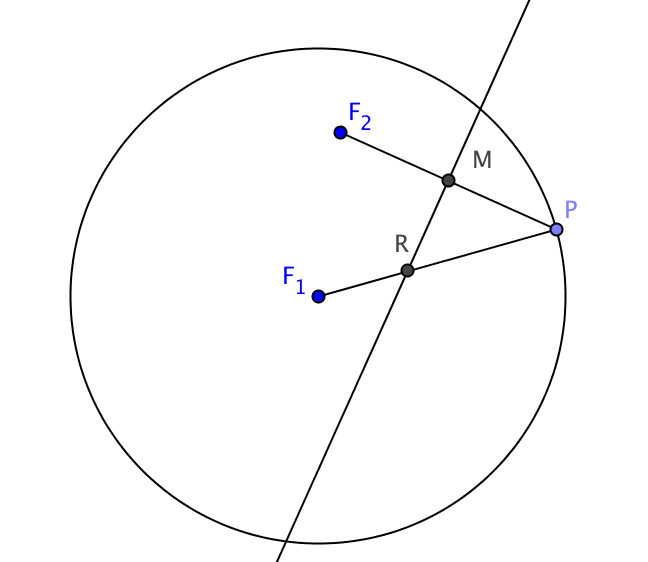
1. Take the wax paper and fold it so that point *P* lies on top of point *F*2. Crease sharply.
2. Repeat step (5) above for all the other points you plotted on the circle (back in step 4).

That is, treat each point on the circle as another “point *P*.” Simply fold each “point *P*” on

the circle to point *F*2. *Be sure to crease sharply each time!*

2. Describe the figure formed by the intersection of all the creases.

3. Recall the definitions of secants and tangents of circles. Apply a similar definition to other closed curves. How would you describe how each of the creases is related to the figure they formed?

We will now prove that the figure is indeed an ellipse with foci at *F*1 and *F*2.

4. The figure at the right shows one of the points *P* that was folded onto *F*2.

This fold line is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ bisector of . It passes through *M*, the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of and intersects radius at point \_\_\_\_\_\_\_.

5. Every point on is equidistant from points and .

This means that *RP* = \_\_\_\_\_.

6. We will now show that the sum of its distances from the two foci to *R* is constant, no matter which point on the circle was chosen as point *P.*

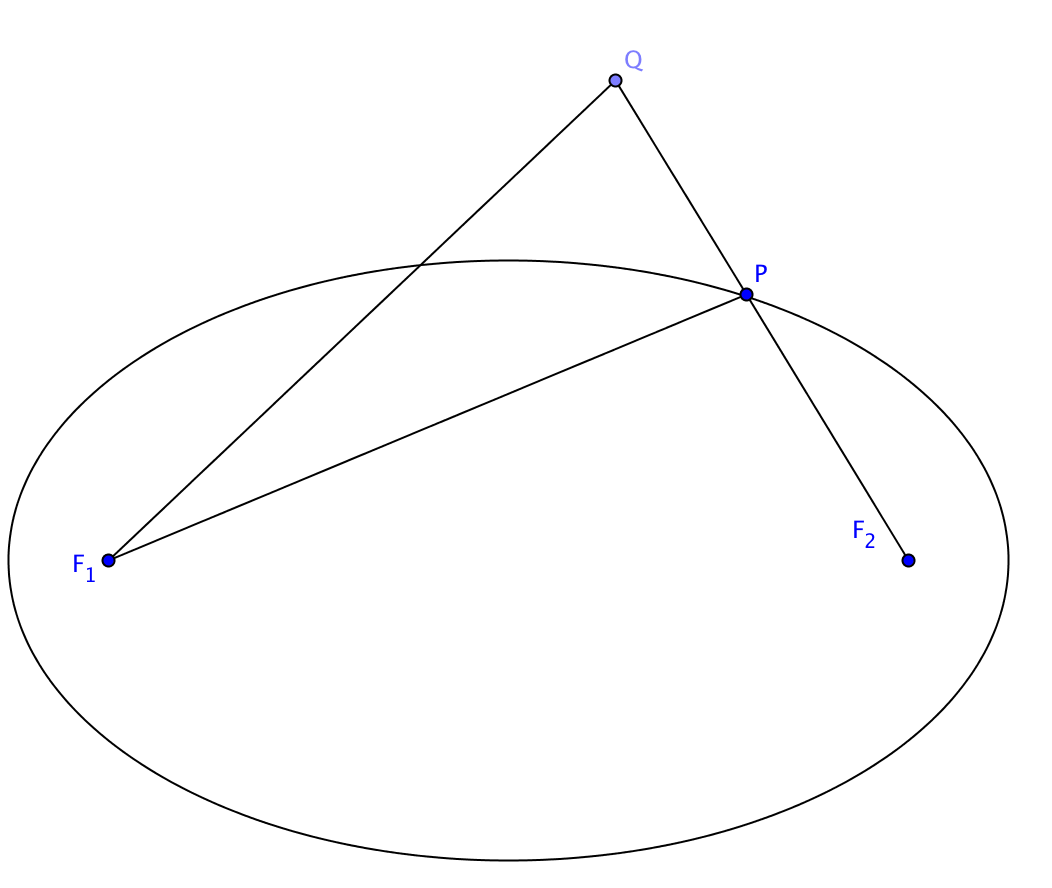
a. Since the radius of a circle *never changes*, it is said to be .

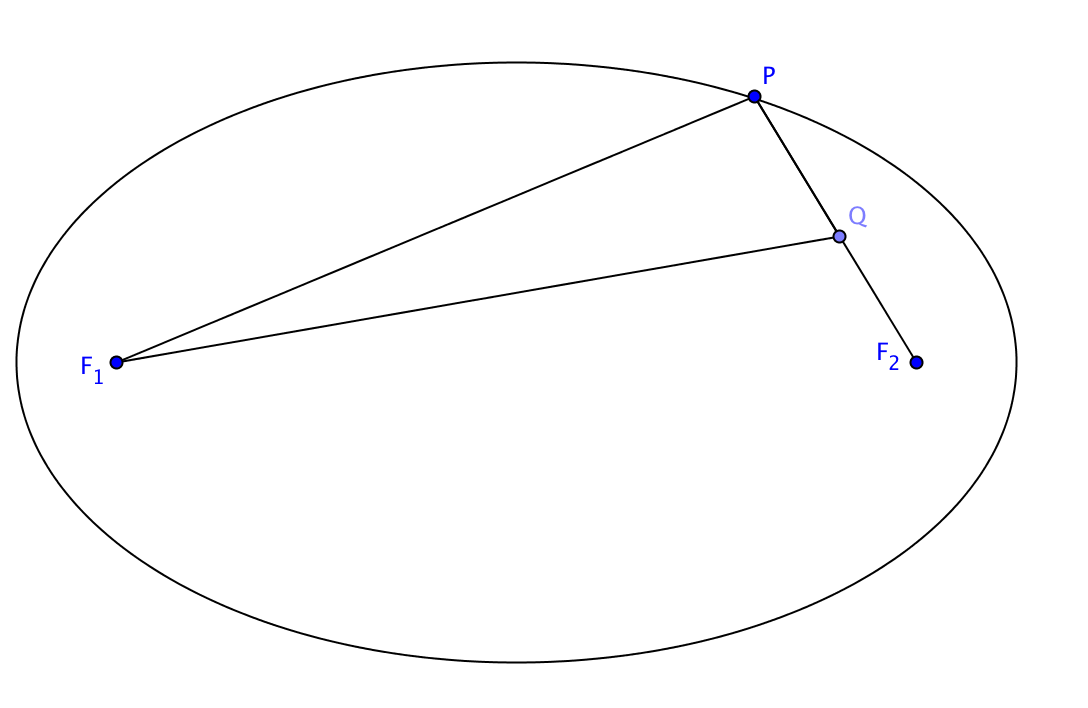
b. By segment addition, the radius *F*1*P* = \_\_\_\_\_ + \_\_\_\_\_\_\_

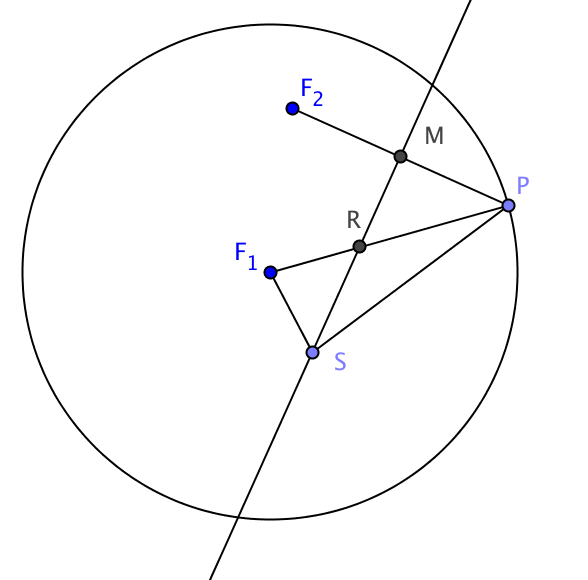
c. But we saw in question 5 that *RP* = \_\_\_\_\_. Therefore *F*1*P* = \_\_\_\_\_ + \_\_\_\_\_\_\_.

d. Therefore *RF1*  + *RF2* is the same for all points *R*. Therefore all points *R* lie on an \_\_\_\_\_\_\_\_\_\_\_ with foci ­­­\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_.

7. It remains to show that each point *R* is the point where its crease is tangent to the ellipse.



1. First we must show that if a point is *outside* an ellipse then the sum of its distances from the two foci is *greater* than the sum of the distances from the foci to a point *on* the ellipse.  
     
   In the figure at the right we have an ellipse with foci at *F*1 and *F*2. Use the Triangle Inequality to prove that   
   *QF*1 + *QF*2> *PF*1 + *PF*2.
2. Use a similar argument to show that if a point is *inside* an ellipse, then the sum of its distances from the two foci is *less* than the sum of the distances from the foci to a point *on* the curve. Use this fact to establish the converse of the statement in part (a).

1. Now go back to our original construction. *R* is the point where the perpendicular bisector of

intersects radius . Let *S* be any other point on the crease.   
  
Show that *SF*1 + *SF*2 > *RF*1 + *RF*2.

1. Our conclusion is that *S* must be \_\_\_\_\_\_\_\_\_\_\_(inside outside, or on?) the curve.
2. But *S* was any point the crease except for *R*. Therefore and the ellipse have only one point in common, so is \_\_\_\_\_\_\_\_\_\_\_\_ to the ellipse