**Activity 2.4.2 Zeros? Real or Complex? How Do We Know?**

Sometimes we need to know how many zeros or x-intercepts a function will have before we graph the function. When the function is given in the Vertex Form: *f(x) = a(x–h)2 +k*, we can make that determination based on the values of *a, h*, and *k*.

1. Determine how many x-intercepts each of the following equations will have based on the Vertex Form of the quadratic equation. Explain how you made your decision.

a. f(x) = –2(x+1)2 + 5

b. g(x) = 0.5(x– ¾)2

c. h(x) = 2.5(x–3.4)2 + 7.4

If all quadratics were in Vertex Form, the determination of the number of x-intercepts could be by inspection and depends on the value of *a* to determine the concavity of the parabola and the value of *k* to determine the y-coordinate of the vertex. However, most quadratics are given in Standard Form: *f(x) = ax2 + bx + c.* We would still like to know how to determine the number of x-intercepts prior to graphing the functions. One way would be to put the function in Vertex Form, but that would require using the completing the square method, which could take time and possibly produce errors.

Instead we can find the number of x-intercepts by examining the zeros of the function found using the quadratic formula:

$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$

 We know that there will be no x-intercepts when there are no real solutions to the quadratic equation *ax2 + bx + c = 0.* To explore this relationship, fill in the table below and answer the questions.

Work with a partner to find the zeros of the following quadratic functions with rational coefficients tosee how many zeros there are and the nature of those zeros. In addition, determine whether or not the equation could be solved by factoring. For each problem determine the value of *b2 – 4ac*.

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| **Function** | ***b2–4ac*** | **Number of Real Zeros** | **Nature of Zeros:****Complex or Real** | **Actual Zeros** | **Factors****If****Possible** |
| *f(x)=3x2 – 2x – 1* |  |  |  |  |  |
| *g(x)=x2 + 6x + 9* |  |  |  |  |  |
| *h(x)=-x2+4x–7* |  |  |  |  |  |
| *k(x)=–.5x2+4x* |  |  |  |  |  |
| *m(x)=5x2+2x+8* |  |  |  |  |  |

2. Examine the results and fill in the statements below with your findings.

When *b2–4ac > 0,* then

When *b2–4ac = 0,* then

When *b2–4ac < 0,* then

Why do you think we call the expression *b2–4ac*  the **Discriminant**?

1. We can make some other observations based on the value of *b2–4ac* for quadratic functions with rational coefficients*.*
2. When will the function have rational roots?

1. When will the function be factorable?

1. When will the graph of the function be tangent to the x-axis?

d. When will the graph of the function have no x-intercepts?