**Activity 5.2.3 The Remarkable e**

1. It can be shown that *e* $≈1+ \frac{1}{1}+\frac{1}{1\left(2\right)}+\frac{1}{1\left(2\right)\left(3\right)}+\frac{1}{1\left(2\right)\left(3\right)\left(4\right)}+ \frac{1}{1\left(2\right)\left(3\right)\left(4\right)\left(5\right)}+\cdots $

The more terms you add together the better the approximation.

*e* is approximately \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ according to your technology.

1. Add the 5 terms shown above using technology. Keep 9 decimal places. \_\_\_\_\_\_\_\_ How many agree with *e*? \_\_\_\_\_
2. Add the first 8 terms \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Keep 9 decimal places. \_\_\_\_\_\_\_\_ How many agree with *e*? \_\_\_\_\_
3. Keep adding terms until you agree to 7 decimal places. How many terms did you need?
4. Keep adding terms until you agree to 8 decimal places. How many terms did you need?
5. Use technology to graph y = 2x. Then zoom in centered around the y-intercept (0, 1) until your curve looks like a line. Trace on your line and select a point other than (0,1). Now use the y-intercept and the point you selected to compute the slope of the “line.” You should get about .69. If you did not, try again.
6. Use technology to graph y = 3x. Then zoom in centered around the y-intercept (0, 1) until your curve looks like a line. Trace on your line and select a point other than (0,1). Now use the y-intercept and the point you selected to compute the slope of the “line.” You should get a slope number about 1.09. If you did not, try again.
7. Now try a base between 2 and 3 Such as y = 2.5x and compute the slope. Continue with trial and error until the slope you get is about 1 using the base. What number worked?\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
8. Now graph y = *e*x. Select a point such as (1, *e*). Zoom in on the point till the graph appears linear close to close to the point (1, *e*). Locate another point on the line and compute the slope . Did you get a number close to *e*?\_\_\_\_\_\_\_
9. Repeat but now use the point (2, *e*2). When you computed your slope did it come close to a decimal approximation for *e*2? \_\_\_\_\_\_\_\_\_ If you did not try another point after zooming in closer and recompute the slope.
10. Now graph y = x2. Select a point such as (1,4). Zoom in as you did in parts A and B and computer the slope. Did you get a number close to 4?\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The function f(x) = k*e*x is the only function whose “slope” at a point (a, f(a)) is equal to kf(a) for all Real numbers. Parts A – C of this problem should suggest this observation. Of course you can try other points on the graphs of y = *e*x andy = x2 to convince you further. For those of you who take calculus, you will be able to prove this observation.

1. For Fun: Cut Up and Then Multiply
2. Take a number such as 36 and divide it up into equal parts say 4 parts. Then 36/4 = 9 Each part will now be 9 in size. 9∙9∙9∙9 = 94 = 6,561

Now try 6 pieces of equal size so 36 /6 = 6 so each part will be 6 and 66 = 46,656 even bigger. How big can we get? Try making the parts as close to *e* as you can.

 Try 12 pieces of equal size. 36/12 = 3 and 312 = 531,441

 Try 14 pieces 36/14 is about 2.6 and 2.614 = 645,099.7.

 Note 2.6 is quite close to *e*.

1. You try it. Try with 10 and cut 10 into 3 pieces, then 4 pieces, then 5 pieces. 10/4 = 2.5 which was closest to *e* for the 3 trials we made. I bet that gave you the largest power.
2. Just an interesting note. If you have studied some statistics you may have heard of a bell or normal curve. Its equation is f(x) = $\frac{e^{-.5x^{2}}}{√2π}$.
3. Another interesting note. The constant for computing distances on a Mercator map uses the constant 180/(π log *e*).
4. If you want to approximate *e* by a quotient and keep the numbers in the numerator small (under 1000) then 878/323 will do the trick. Divide and see how many places of agreement you get. \_\_\_\_\_
5. One last tidbit. You can remember e to 15 decimal places by 2.7 1828 1828 45 90 45. The last 6 digits think isosceles right triangle !!!!!