**Activity 5.2.1 How Many Compounding Periods Should I Try to Get?**

In unit 7 of Algebra 1 you examined compound interest on a bank savings account. You found that you will get more interest if you compound more frequently. Let us review. Suppose you can get 12% interest. That is not possible now in a bank but it will keep our computations a bit simpler. Let us pretend our parents have won a big state lottery and you want to help them invest the money. So let us pretend your parents won 10 million dollars. We will be solving a few problems and to look for patterns, it will be helpful to organize our work in a table. We will include a column for simple interest too just to remind us that with simple interest we do not earn interest on interest.

1. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank that offered 12% interest compounded once a year.

1. How much money will you have after a year? \_\_\_\_\_\_\_\_\_\_\_Place your answer in the table found in problem 9.
2. How much simple interest will you earn? \_\_\_\_\_\_\_\_What will be the amount in the bank if you went with a bank promising 12% simple interest.\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Now let the money stay in the bank promising 12% compound interest once a year for 5 years. \_\_\_\_\_\_\_\_\_\_\_\_\_\_ Compare with the bank that promises 12% simple interest for the 5 years. \_\_\_\_\_\_\_\_\_\_\_
4. If you left your money in each bank for t years, what functions (for simple and compound) would you use to find the amount in the bank after t years? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank promising 12% compounded twice a year.
2. How much money will you have after a year?\_\_\_\_\_\_\_\_ Place your answer in the table in number 9.
3. How much simple interest will you earn?

What will be the amount in the bank if you went with a bank promising 12% simple interest? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Now let the money stay in the bank promising 12% compound interest twice a year for 5 years. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Compare with the bank that promises 12% simple interest for the 5 years.\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. If you leave your money in each bank for t years, what functions would give you the amount you would have after t years?
3. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank promising 12% compounded 4 times a year.
4. How much money will you have after a year? Place your answer in the table in number 9. \_\_\_\_\_\_\_\_\_\_\_\_\_Did you make a lot more interest than you did with the once or twice a year compounding? \_\_\_\_\_ Explain.
5. How much simple interest will you earn? \_\_\_\_\_\_\_\_What will be the amount in the bank if you went with a bank promising 12% simple interest. \_\_\_\_\_\_\_\_\_\_\_\_
6. Now let the money stay in the bank promising 12% compound interest quarterly for 5 years. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Compare with the bank that promises 12% simple interest for the 5 years.\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
7. If you leave your money in each bank for t years, what functions would give you the amount you would have after t years?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank promising 12% compounded 12 times a year.

1. How much money will you have after a year? \_\_\_\_\_\_\_\_\_\_\_\_\_ Place your answer in the table in number 9. Did you make a lot more interest than you did with the once or twice a year compounding?\_\_\_\_\_\_\_ Explain

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. How about how much simple interest will you earn? \_\_\_\_\_\_\_\_\_\_What will be the amount in the bank if you went with a bank promising 12% simple interest. \_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Now let the money stay in the bank promising 12% compound interest monthly for 5 years. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Compare with the bank that promises 12% simple interest for the 5 years.\_\_\_\_\_\_\_\_\_
3. If you leave your money in each bank for t years, what functions would give you the amount you would have after t years? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank promising 12% compounded 365 times a year.

1. How much money will you have after a year? $\_\_\_\_\_\_\_\_\_\_\_\_Place your answer in the table in number 9. Did you make a lot more interest than you did with the once or twice a year compounding?\_\_\_\_\_\_\_\_. Explain.
2. How about how much simple interest will you earn? \_\_\_\_\_\_\_\_\_\_\_What will be the amount in the bank if you went with a bank promising 12% simple interest. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Now let the money stay in the bank promising 12% compound interest daily for 5 years. $\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Compare with the bank that promises 12% simple interest for the 5 years.\_\_\_\_\_\_\_\_\_\_
4. If you leave your money in each bank for t years, what functions would give you the amount you would have after t years?\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank promising 12% compounded every hour.
6. How much money will you have after a year? \_\_\_\_\_\_\_\_\_\_\_Place your answer in the table below.
7. How about how much simple interest will you earn? \_\_\_\_\_\_\_\_\_\_What will be the amount in the bank if you went with a bank promising 12% simple interest. \_\_\_\_\_\_\_\_\_\_\_\_\_
8. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank promising 12% compounded every minute.
9. How much money will you have after a year?\_\_\_\_\_\_\_\_\_\_\_\_ Place your answer in the table below.
10. Fill in the remainder of the row.
11. Suppose you convince your parents to invest the 10 million dollars (after taxes) in a bank promising 12% compounded every second.
12. How much money will you have after a year?\_\_\_\_\_\_\_\_\_\_ Place your answer in the table below.
13. Fill in the remainder of the row.
14. Examine your table. Does it make much difference if you compound 12 or 365 or every minute or second? \_\_\_\_\_\_\_\_\_\_\_\_\_\_Explain what is happening? \_\_\_\_\_\_\_

|  |  |  |
| --- | --- | --- |
| Frequency of Compounding | Amount at the End of one year | Simple Interest at the end of one year |
| Once a year |  |  |
| Semiannually |  |  |
| Quarterly |  |  |
| Monthly |  |  |
| Weekly |  |  |
| Daily |  |  |
| Hourly |  |  |
| Every Minute |  |  |
| Every Second |  |  |
| Continuously |  |  |

1. Now use you calculator to find *e* (over ln on a TI). What decimal approximation do you get \_\_\_\_\_\_\_\_\_\_\_\_ You have found another number that is like π. You have found another irrational number whose decimal expansion does not repeat nor does it terminate (even though it looks like it might).
2. You were using the formula A = P ( 1 + 0.12/n)nt to obtain your values in the first column above. Of course t is one for number 9 above. Let n be very large say 10,000,000 and compute 10000000(1 + 0.12/n)n. \_\_\_\_\_\_\_\_\_\_\_Compare that value to 10000000e0.12 . \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

When banks promise continuous compounding they are using the formula A = P*e*rt. P is the amount invested, r is the annual rate of interest promised and t is the number of years.

1. Fill in the last row of the table in number.
2. Did your parents make more money with continuous compounding versus compounding every second? \_\_\_\_\_\_\_\_\_\_\_\_\_

.

1. We do not know who, but someone noticed the curious fact that if a principal amount is compounded many times a year for t years that the amount of money approaches a certain amount or limit—that compounding more often does not increase the amount of money. Let us see what this limit is for one dollar. Suppose you will get a rate of 100% and your principle is $1. Fill in the table below in number 15. It appears if *n* is very large $1(1 + 1/n)^{n}$ = \_\_\_\_\_\_\_\_\_\_You have discovered that each $1 when compounded continuously at an annual rate of 100% is worth $2.718 to 3 decimal places. The formula A = P*e*rt allows us to compound continuously for rates other than 100%, useful in many applications as you will see in future activities.

|  |  |  |
| --- | --- | --- |
| Frequency of Compounding | Amount at the End of one year | Simple Interest at the end of one year |
| Once a year |  |  |
| Semiannually |  |  |
| Quarterly |  |  |
| Monthly |  |  |
| Weekly |  |  |
| Daily |  |  |
| Hourly |  |  |
| Every Minute |  |  |
| Every Second |  |  |
| 100 million |  |  |
| Continuously |  |  |