**Unit 3 Investigation 6 (3 Days)**

**Polygons with Coordinates**

**Common Core State Standards**

* G-CO.10. Prove theorems about triangles. *Theorems include... the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length, the medians of a triangle meet at a point.*
* G-CO.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent , ..., the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*
* G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle...*

**Overview**

In this investigation students will use coordinate methods to prove theorems. Many theorems from Investigation 5 that were proved using “synthetic” methods will be reproved in this investigation using coordinates. Students will be asked to compare coordinate and synthetic methods of proof.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Prove properties of quadrilaterals and triangles given numerical coordinates for the vertices.
* Prove properties of quadrilaterals and triangle given variable coordinates for the vertices.
* Apply properties of quadrilaterals.
* Find midsegments of trapezoids and triangles
* Find centroids of triangles.

**Assessment Strategies: How Will They Show What They Know?**

* + In **Exit Slip 3.6.1** students will classify a quadrilateral with specific coordinates.
	+ **Exit Slip 3.6.2** will require students to find the median of a trapezoid given the lengths of the bases.
	+ **Journal Entry** will ask students to explain how coordinate proof is different from synthetic proof.

**Launch Notes**

Discuss Rene Descartes the inventor of coordinate geometry.

Here are three videos you may want to use. Descartes Paradise Algebra Rap[www.youtube.com/watch?v=mFYJu9kRJVU](http://www.youtube.com/watch?v=mFYJu9kRJVU) runs for just over 3 minutes. It alludes to the story that Descrates got his idea by watching a fly move about on the ceiling. A longer video that retells that story is found at [www.youtube.com/watch?v=0KfKhvJLH1E](http://www.youtube.com/watch?v=0KfKhvJLH1E).

Descartes and Cartesian Coordinates [www.**youtube**.com/watch?v=N4nrdf0yYfM](http://www.youtube.com/watch?v=N4nrdf0yYfM) runs for eleven minutes, but provides an excellent review of the coordinate plane for students who need a refresher.

Descartes’ invention of coordinate geometry was a major accomplishment. It paved the way for more advanced mathematical topics like calculus. Software such as Geogebra and GSP uses coordinates to locate points.

In this investigation we will use the methods of coordinate geometry to prove some of the properties of quadrilaterals that were introduced in the previous investigation. Most of the proofs we have seen so far in this course have not used coordinates. They are called “synthetic” proofs as opposed to the coordinate proofs we will see in this investigation.

**Teaching Strategies**

Before starting the first activity you should review the tools that will be used in coordinate proofs:

* Parallel lines have the same slope. (Exception: vertical lines have undefined slope)
* Perpendicular lines have slopes that are opposite reciprocals of each other (exception: horizontal lines with slope = 0 are perpendicular to vertical lines with undefined slope)
* The Distance formula
* The Midpoint formula

In **Activity 3.6.1 Properties of Quadrilaterals** students revisit the hierarchy of quadrilaterals introduced in Investigation 5. They are given the numerical coordinates of the four vertices asked to find the slopes and lengths of the sides. They can then classify the quadrilateral as one of the special quadrilaterals on the chart or as a general quadrilateral. They can also determine whether the diagonals have any of these properties: congruent, perpendicular, or bisecting each other.

**Differentiated Instruction (For Learners Needing More Help)**

These students may have difficulty with variable coordinates. Encourage them to spend more time on **Activity 3.6.1** rather than the next three activities to consolidate their ability to use the slope, distance and midpoint formulas, as long as they understand that in the first activity they are only looking at specific cases and their findings do not constitute a proof of the general theorem. See comment on inductive and deductive reasoning follow **Activity 3.6.2** below.

**Activity 3.6.2 Variables for Coordinates** introduces students to the idea that by using variables, general results can be verified. For example if the coordinates of the vertices are P(0,0), Q(*a*, 0), R(*a*, *c*), and S(0, *c*), it is evident that the opposite sides are parallel and adjacent sides perpendicular. Therefore this quadrilateral is a rectangle. Students then use the distance formula to show that the diagonals are congruent and the midpoint formula to show that the diagonals bisect each other.

 Theorems addressed in Activity 3.6.2 include:

 **Parallelogram Opposite Sides Theorem**

 **Parallelogram Diagonals Theorem**

 **Rectangle Diagonals Theorem**

 **Rhombus Diagonals Theorem** (use (*a*, 0), (0, *c*), (–*a*, 0) and (0, –*c*) for the vertices)

In introducing Activity 3.6.2 it would be good to reemphasize the difference between inductive and deductive reasoning. In Activity 3.6.1 we look at specific cases that support a conjecture. We can prove that the diagonals of a particular rectangle are congruent, but to prove that the conjecture holds for all rectangles, we need to use variables.

**Group Activity**One way to bridge the gap between numerical coordinates and variable coordinates is to have students work in small groups to gather evidence to support a conjecture. For example a general parallelogram may be represented by *P*(0, 0), *Q*(*a*, 0), *R*(*b + a*, *c*) and *S*(*b*, *c*). Have each student in the group pick values for *a*, *c*, and *b*. Have them sketch their parallelograms in the coordinate plane. Then have them find the midpoints of the diagonals $\overbar{PR}$ and $\overbar{QS}$. In every case they should find that the diagonals bisect each other. They have found evidence that supports to Parallelogram Diagonals Conjecture, but it does not prove it in the general case as a theorem. For the general case we need to use variables; and if that is too daunting for the students, the teacher can demonstrate the proof to the whole class.

**Activity 3.6.3 Quadrilaterals in Standard Position** starts with the idea that with translation and rotation any quadrilateral may be transformed so that its image has one vertex at the origin and one side along the positive *x­-*axis. Thus a general quadrilateral may be represented with coordinates *P*(0,0), *Q*(*a*, 0), *R*(*b*, *c*), *S*(*d*, *e*). The GeoGebra activity suggested in question 1 may be assigned to students to do on their own or used for a classroom demonstration by the teacher.

Once the coordinates for a general quadrilateral have been established, we may look at specific cases. For *PQRS* to be a trapezoid with $\overbar{PQ}$ || $\overbar{RS}$ we must have *e* = *c*. For the trapezoid to be a parallelogram we must have *b = a + d*. For the parallelogram to be a rectangle we must have *d* = 0. And finally for the rectangle to be a square we must have *c* = 0. The more special the type of quadrilateral, the fewer variables are required for its representation. Since these proofs are rather abstract, you may want to limit this activity to the more able students.

**Differentiated Instruction (Enrichment)**

Encourage able students to stretch their algebraic skills by working on Activity 3.6.3 and the general case of the Centroid Theorem in Activity 3.6.6.

In **Activity 3.6.4 Converses of Quadrilateral Theorems** students begin with a more general type of quadrilateral and a specific property to show that the quadrilateral is of a more specific type. For example given a quadrilateral with coordinates *P*(0, 0), *Q*(*a*, 0), *R*(*b*, *c*) and *S*(*d*, *e*), if the diagonals bisect each other we have $\left(\frac{b}{2}, \frac{c}{2}\right)=(\frac{a+d}{2},\frac{e}{2})$ from which it follows that *b = a + d* and *e* = *c*. Therefore *PQRS* must be a parallelogram.

Theorems addressed in Activity 3.6.4 include

 **Parallelogram Diagonals Converse**

 **One Pair Congruent and Parallel Theorem**

 **Rectangle Diagonals Converse**

 **Rhombus Diagonals Converse**

These proofs require a fairly high level of algebraic sophistication and may not be suitable for all students.

**Activity 3.6.5 Midsegments** introduces midsegments in triangles and trapezoid. Students use coordinate methods to prove the Trapezoid Midsegment Theorem. To avoid computation with fractions they may want to represent the coordinates of the trapezpoid as P(0,0), Q(2*a*, 0),
R(*2b*, 2*c*) and S(2*d*, 2*c*). The midpoints of the legs then become (*d*, *c*) and (*a* + *b*, *c*). After proving the theorem they consider the special case where *b = d +a* and the figure is a parallelogram. They will observe that the length of the midsegment is the same as the lengths of the two sides to which it is parallel. Another special case occurs when *b = d*. We now have a triangle rather than a trapezoid. This leads to the Triangle Midsegment Theorem, in which the midsegment is half the length of the opposite side. A construction with GeoGebra or GSP can be used to show how the two special cases arise as point R is moved parallel to PQ.

In **Activity 3.6.6 Medians and Centroids** the centroid of a triangle is the defined as the point in the plane whose coordinates are the average of the coordinates of the three vertices. This is an extension of the concept of the midpoint of a segment as the average of the two endpoints. By finding the equations of the three medians of the triangle, we can show that the centroid lies on all three medians, and therefore the medians are concurrent at this point. This is illustrated first with specific coordinates, e.g. P(0, 0), Q (18, 0), and R(12, 24), then with more general coordinates P(0, 0), Q(6*a*, 0), R(6*b*, 6*c*). In the latter case multiples of 6 are used to simplify the computation. Just the same, you may want to assign the general case only to more able students. Students may illustrate the property of the centroid as the center of gravity by balancing a triangle cut out of a file card or stiff paper at its centroid on the end of a pencil. (See appendix for derivations).

**Journal Entry**

Pick a theorem and show how it can be proved both by synthetic and coordinate methods. Look for students to show an understanding of both proofs.

**Closure Notes**

Have students reflect on the difference between synthetic and coordinate methods of proof. Pick a particular theorem (e.g. Parallel Opposite Sides Theorem), look at the two proofs side by side and compare. Ask students which proof they prefer and why.

**Vocabulary**

Coordinate proof

Synthetic proof

Base (of Trapezoid)

Leg (of Trapezoid)

Midsegment (of Trapezoid, triangle)

Median (of Triangle)

Centroid (of Triangle)

Concurrent lines

**New Theorems** (in addition to those introduced in Investigation 5)

**Trapezoid Midsegment Theorem:** The segment joining the two legs of a trapezoid is parallel to the bases and equal in length to the average of the lengths of the two bases.

**Triangle Midsegment Theorem:** The segment joining two sides of a triangle is parallel to the third side and equal in length to half the third side.

**Centroid Theorem:** The three medians of a triangle are concurrent at the centroid.

**Resources and Materials**

Activity 3.6.1 Properties of Quadrilaterals

Activity 3.6.2 Variables for Coordinates

Activity 3.6.3 Quadrilaterals in Standard Position (may not be done by all students)

Activity 3.6.4 Converses of Quadrilateral Theorems (may not be done by all students)

Activity 3.6.5 Midsegments

Activity 3.6.6 Medians and Centroids

Graph paper

Rulers

Stiff paper or file cards

Scissors

Geogebra or GSP