**Activity 3.6.2 Variables for Coordinates**

****

In this activity you will use variable coordinates to prove theorems using coordinate geometry.

1. Prove the **Parallelogram Opposite Sides Theorem:** The opposite sides of a parallelogram are congruent. (If a quadrilateral is a parallelogram, then the opposite sides are congruent.)

Here is a quadrilateral in the coordinate plane with vertices *P*(0, 0), *Q*(*a,* 0), *R*(*a +b*, *c*), and *S*(*b*, *c*).

First show that the opposite sides are parallel (using the slope formula):

 a. Show that $\overbar{PQ}$ || $\overbar{RS}$

 b. Show that $\overbar{QR}$ || $\overbar{SP}$

c. Since both pairs of opposite sides are parallel, by definition *PQRS* is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Now show that the opposite sides are congruent (using the distance formula):

 d. Show that *PQ* = *RS*

e. Show that *QR* = *SP*

 f. Since *PQ* = *RS* and *QR* = *SP*, the opposite sides of *PQRS* are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

2. Prove the **Parallelogram Diagonals Theorem:** The diagonals of a parallelogram bisect each other. (If a quadrilateral is a parallelogram, the diagonals bisect each other.)

You already showed in question 1 that quadrilateral in the coordinate plane with vertices *P*(0, 0), *Q*(*a,* 0), *R*(*a +b*, *c*), and *S*(*b*, *c*) is a parallelogram.

1. Find the midpoint of $\overbar{PR}$: (\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_)

 b. Find the midpoint of $\overbar{QS}$: (\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_)

 c. Since diagonals $\overbar{PR}$ and $\overbar{QS}$ have the same \_\_\_\_\_\_\_\_\_\_, they\_\_\_\_\_\_\_\_\_\_ each other.

3. Prove the **Rectangle Diagonals Theorem:** The diagonals of a rectangle are congruent. (If a quadrilateral is a rectangle, then its diagonals are congruent.)

Here is a quadrilateral in the coordinate plane with vertices *P*(0, 0), *Q*(*a,* 0), *R*(*a, c*), and *S*(0, *c*).

1. Which sides have zero slopes?
2. Which sides have undefined slopes?
3. Explain, using slope why $\overbar{PS }⊥$ $\overbar{PQ}$.
4. Name the other pair of perpendicular sides.
5. Explain why *PQRS* must be a rectangle.

Now find the length of each diagonal:

1. *PR* = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 g. *QS* = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 h. Since *PR* = *QS*, the diagonals of PQRS are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

4. Prove the **Rhombus Diagonals Theorem:** The diagonals of a rhombus are perpendicular. (If a quadrilateral is a rhombus, then its diagonals are perpendicular.)

Here is a quadrilateral in the coordinate plane with vertices *P*(*a*, 0), *Q*(0, *c*), *R*(–*a,* 0), and *S*(0, –*c*).

Show that *PQRS* is a rhombus (use the distance formula):

 a. *PQ* = \_\_\_\_\_\_\_\_\_

 b. *QR* = \_\_\_\_\_\_\_\_\_

 c. *RS* = \_\_\_\_\_\_\_\_\_

 d. *SP* = \_\_\_\_\_\_\_\_\_

 e. Because all four sides are congruent, *PQRS* is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

 f. Find the slope of diagonal $\overbar{PR}$.

 g. Find the slope of diagonal $\overbar{QS}$.

 h. Explain using slopes why $\overbar{PR}$ $⊥$ $\overbar{QS}$.