**Activity 3.5.4 Properties of Parallelograms**

You have been investigating properties of the different quadrilaterals. In this activity we will prove some properties of parallelograms.

First let’s review the definition and properties that we investigated already.

**1.** A parallelogram is defined as a quadrilateral with two pairs of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ sides.

**2.** Earlier in this investigation you may have discovered these properties:

a. The opposite sides of a parallelogram are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

b. The opposite angles of a parallelogram are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

c. The diagonals of a parallelogram \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ each other.

Now, let’s prove the properties as they follow from the definition of parallelogram.

3. **Parallelogram Opposite Sides Theorem:** If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Fill in the blanks to complete the proof. Mark pairs of congurent sides and angle on the diagram.

Given: *ABCD* is a parallelogram.

Prove: $\overbar{AB} ≅ \overbar{CD}$ and $\overbar{AD} ≅ \overbar{CB}$

Draw diagonal $\overbar{AC}$

Since ABCD is a parallelogram,  and \_\_\_\_\_\_\_\_\_\_\_\_. Since , \_\_\_\_\_\_\_\_ because Alternate Interior Angles formed by parallel lines are congruent.

Since \_\_\_\_ || \_\_\_\_\_, \_\_\_\_\_\_\_because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

We also know that \_\_\_\_\_\_  because $\overbar{AC}$ is a shared side.

∆ *ABC* $≅∆$\_\_\_\_\_ by the ASA Congruence Theorem.

Therefore,  and \_\_\_\_\_\_\_\_\_\_\_\_\_\_ because CPCTC.

4**.** **Parallelogram Opposite Angles Theorem:** If a quadrilateral is a parallelogram, then its opposite angles are congruent.

Write a proof of the parallelogram opposite angles theorem as it follows from the definition of parallelogram.

Hint: Draw diagonal $̿$ to prove $∠ABC ≅ ∠ADC $. Then draw diagonal $̿$ to prove that $∠DAB ≅ ∠DCB$.

**5.** **Parallelogram Diagonals Theorem:** If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Fill in the blanks to complete the proof. Mark pairs of congruent sides and angles on the diagram.

E

Given: *ABCD* is a parallelogram with diagonals $\overbar{AC}$ and $̿$ intersecting at *E*.

Prove: $\overbar{AE}≅\overbar{CE} $and $\overbar{DE}≅\overbar{BE}.$

Since *ABCD* is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,  by definition of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

It follows that \_\_\_\_\_\_\_ and $∠ABD$ $≅$\_\_\_\_\_\_\_\_\_\_ because Alternate Interior Angles formed by parallel lines are congruent.

Since opposite sides of parallelograms are congruent, \_\_\_\_\_\_\_\_.

It follows that ∆ *AEB*$ ≅$ ∆\_\_\_\_\_\_\_\_\_ by \_\_\_\_\_\_\_\_\_\_.

Thus,  and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ by CPCTC. Therefore, the diagonals of *ABCD* bisect each other by definition of bisect.