**Activity 3.4.2 Pascal’s Pizza Parlor!**

We did the pizza problem to find how many different pizzas we could make with 5 toppings. The solution to the problem found that there were 32 possible pizzas with 5 toppings. However, there are some interesting patterns that can be observed when you consider the number of different pizzas you could make if you had no toppings, 1 topping, 2 toppings, 3 toppings, 4 toppings, etc. The table below can help you organize your exploration. The rows of the table indicate the number of toppings that you could have: 0, 1, 2, …. (Assume all pizzas have cheese to start.) The columns indicate how many different kinds of pizzas you could make with that many toppings. For instance, if you had no toppings you could only have a cheese pizza, so in the first row you would put a 1 in the column with 0 toppings and nothing in the rest of the columns. If you had 1 topping you could make 1 pizza with no toppings and 1 pizza with the 1 topping. The rest of the columns would be empty.

1. Now complete the table by listing all the different pizzas you could make when you have 0, 1, 2, 3, 4, 5, or 6 toppings. The first two rows and the row for 5 toppings are filled in already as described above. When there are 2 or more toppings to choose from, find all the different pizzas you can make for the different number of toppings you can put on the pizza. Then fill in the values for rows 3 and 4. When those are done answer the questions.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| No. of Toppings | Pizzas with 0 toppings | Pizzas with 1 topping | Pizzas with 2 toppings | Pizzas with 3 toppings | Pizzas with 4 toppings | Pizzas with 5 toppings | Pizzas with 6 toppings | Total # of toppings |
| 0 | 1 |  |  |  |  |  |  | 1 |
| 1 | 1 | 1 |  |  |  |  |  | 2 |
| 2 |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 |  | 32 |
| 6 |  |  |  |  |  |  |  |  |

1. Identify a pattern that would allow you to fill in the table for 6 toppings without actually counting all the possible pizzas and then fill in the rows.
2. Find a pattern to determine the total number of pizzas you can make given the number of toppings. Predict how many pizzas you could make with 10 toppings.
3. Using the pattern from #2, predict how many 4 topping pizzas you could make if you had 7 toppings to choose from.

If you arrange the numbers that you found in the table in the shape of a triangle as shown below, you have what is called Pascal’s Triangle. Although it was known well before his time, Blaise Pascal wrote a treatise published in 1665 that developed many applications involving this array of numbers. The first 7 rows of Pascal’s Triangle are listed below.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

From the pizza problem, we see that each different pizza is a subset of the toppings from which you can choose. Each line in the table describes the number of subsets that you can form with 0, 1, 2, . . . , n-1, n elements from a set of n elements where n is the second entry in the line. So for a set of 6 elements, the line in Pascal’s triangle is: 1 6 15 20 15 6 1. Therefore, there is 1 subset with 0 elements, 6 subsets with 1 element, 15 subsets with 2 elements, 20 subsets with 3 elements, 15 subsets with 4 elements, 6 subsets with 5 elements, and 1 subset with all 6 elements.

Using Pascal’s Triangle, answer the following questions. Extend the triangle to as many rows as needed to answer the problems.

1. A basketball team has 8 players. How many ways can you choose the 5 players to start the game, assuming positions don’t matter?
2. The new lottery game is called Pick Four. You have to choose four numbers from the digits 0, 1, 2, •••, 9. How many different ways can you pick the four numbers if order doesn’t matter? Use that number to predict the probability that you will win if you purchase one ticket.
3. In a regular hexagon, how many different triangles could you create using three of the vertices of the hexagon as the vertices of your triangle?