**Unit 2: Investigation 1 (2 Days)**

**Identifying Congruent Figures**

***Common Core State Standards***

* G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
* **G-CO.7**. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

**Overview**

Two figures are congruent if one is the image of the other under a sequence of rigid motions. As a consequence, students will discover that (1) two segments with the same length are congruent, (2) two angles with the same measure are congruent, and (3) two circles with equal radii are congruent. They will also identify corresponding parts of congruent polygons and conclude that two triangles are congruent if and only if all 6 pairs of parts are congruent. Given two congruent figures they will find a sequence of rigid motions mapping one onto the other.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Apply the properties of congruent segments, angles, and circles
* Identify a sequence of rigid motions that maps one of two congruent figures onto another.
* Identify corresponding parts of congruent polygons.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit slip 2.1.1** asks students to apply the properties of congruent segments and angles.
* **Exit slip 2.2.2** asks students to identify corresponding parts of congruent polygons.
* **Journal Entry** asks students to explain the meaning of congruence.

**Launch Notes**

Begin by discussing the importance of interchangeable parts in manufacturing. You might discuss the pioneering role of Connecticut resident Eli Whitney who is largely credited with bringing this innovation to the United States. This 3 ½ minute video tells about Whitney’s invention of the cotton gin and the use of interchangeable parts in the manufacture of guns:

<https://www.youtube.com/watch?v=sLb9QGWK8TA>. For a historical analysis demonstrating that Whitney’s innovation was not original, see <http://www.uh.edu/engines/epi1252.htm>. You may want to consult with a history teacher at your school to learn how this topic is addressed in the social studies curriculum.

Point out that the concept of congruence is the key geometric idea behind interchangeable parts. In this unit we will learn how to identify congruent figures and apply their properties.

**Teaching Strategies**

Ask students how we can tell if two figures are congruent. They are likely to respond that they have the same size and the same shape. What does that mean? Introduce this more precise definition.

**Definition:** Two figures are **congruent** if and only if one is the image of the other under an isometry.

**Activity 2.1.1** has students finding the transformation with paper and pencil tools: rulers, protractors and graph paper. This activity can also be done with Geogebra or Geometer’s Sketchpad. This is shown in the power point slide show **CongruentSegments&Angles.pptx.**

Start with two segments. Students may readily agree that they are the same size if they have the same length. We measure length in linear units such as inches, feet, centimeters, meters, etc. In the coordinate plane we can use the distance formula. Have students draw two segments $\overbar{AB}$ and $\overbar{CD}$ with the same length (using a ruler) and then describe the transformations that map one onto the other. They should find that some combination of translations and rotations will map $\overbar{AB}$ onto $\overbar{CD}$. Remind students that the Isometry Postulate (from Unit 1) says that isometries preserve distance.

Next look at what happens when two angles have the same measure.

Have students start with $∠$ *CAB*. and use a protractor to draw a second angle $∠$*EDF* with the same measure. Ask students to describe transformation that will map $∠$ *CAB* onto $∠$*EDF.* They should find that some combination of translations, rotations, and reflections that will do this. Remind students that the Isometry Postulate (from Unit 1) says that isometries preserve angle measure.

**Differentiated Instruction (Enrichment)**

Challenge students to use one of the software packages, Geogerba or Geometer’s Sketchpad to duplicate the above demonstrations showing congruence of segments and angles. Ask them to show at least two sets of transformations that produce the same result. For example if $\overbar{AB}≅\overbar{CD}$ then C can be mapped onto A and D mapped onto B, or C can be mapped onto B and D mapped onto A. Similarly two congruent angles can be mapped onto each other in two ways, one of which will require a reflection.

Following the discussion above and the student activity, ask the class to draw conclusions about conditions for two segments or two angles to be congruent.

They should arrive at the following:

 If two segments have the same length, then they are congruent.

 If two angles have the same degree measure, then they are congruent.

Then ask the students to consider the converses of these two statements by asking:

 Suppose two segments are congruent, what do you know about their lengths?

 Suppose two angles are congruent, what do you know about their measures?

Since isometries preserve distance and angle measure, the answers should be obvious.

 If two segments are congruent, then they have the same length.

 If two angles are congruent, then they have the same degree measure.

At this point, however, not all students will appreciate the distinction between a conditional statement and its converse. We will be developing that idea throughout this and the following unit. Our objective is to get them to understand the equivalence between statements about congruence and statements about measures.

Here we should introduce the symbol for congruence $≅$ and model its appropriate use.

For segments we write $\overbar{AB}≅\overbar{CD}$ to indicate that two segments are congruent and *AB = CD* to indicate that their lengths are equal. Point out to students that the former is a statement about geometric figures and the latter a statement about two quantities. It is common for students to confuse these and write statements like $\overbar{AB}=\overbar{CD}$ or $AB≅CD$. Explain to students that while these statements convey the idea, they are mathematically incorrect. This is an opportunity to emphasis precision in the use of symbols as well as language.

Similarly with angles we write $∠ABC≅∠DEF$ to indicate that two angles are congruent, but

$m∠ABC= m∠DEF$ to show the equality of their angle measures. Again, the former is a statement about geometric figures and the latter a statement about quantities.

Because the above statements about congruence and equality are logically equivalent you may want to introduce the symbol for logical equivalence, the double “if and only if” arrow. Then we can write:

$\overbar{AB}≅\overbar{CD}$ $⟺ $*AB = CD*,and

$∠ABC≅∠DEF$ $⟺ m∠ABC= m∠DEF$.

**Activity 2.1.2** gives students the opportunity to practice using this new notation. They will also discover that like equality, the congruence relation is reflective, symmetric, and transitive.

Having examined segments and angles, the next question you pose for students should be fairly straightforward: Under what conditions are two circles congruent? They should easily conclude that two circles are congruent if and only if their radii are congruent (or equivalently that their radii have the same length.)

At the end of the first day you may want to give **Exit Slip 2.1.1** which assesses student understanding of congruent segments and congruent angles.

Next we discuss congruent polygons. You may begin this discussion on the second day, or if time permits, toward the end of the first day.

Although our focus will be congruent triangles, begin by discussing polygons in general. Students should be familiar with the term *polygon* and be able to classify a polygon by the number of its sides. For our purposes we can use these definitions:

**Polygon:** a figure formed by three or more line segments (called **sides**) such that each side intersects exactly two other sides, one at each endpoint and no two sides with the same endpoint a collinear. The endpoints of the sides are called **vertices** (singular **vertex**).

**(Interior) angle of a polygon:** the angle formed by rays containing two adjacent sides of the polygon. The phrase “angle of a polygon” refers to an interior angle of the polygon.



We use the word “parts” to describe the interior angles and the sides of a polygon. Thus pentagon ABCDE has ten parts: sides $\overbar{AB}$, $\overbar{BC}$, $\overbar{CD}, \overbar{DE}$, and $\overbar{EA},$ and angles $∠EAB$, $∠ABC$, $∠BCD$, $∠CDE$, and $∠DEA$. In the absence of other lines through the vertices we can name the angles with one letter as

$∠A$, $∠B$, $∠C$, $∠D$, and $∠E$.

Pentagon KLMNO is the image of ABCDE under a translation followed by a rotation. Therefore, by definition, the two pentagons are congruent. Since rigid motions preserve distance and angle measure we can conclude that the corresponding parts of the two polygons are congruent as well, namely:

Corresponding sides: $\overbar{AB}≅\overbar{KL}$, $\overbar{BC}≅\overbar{LM}$, $\overbar{CD}≅\overbar{MN}$, $\overbar{DE}≅\overbar{NO}$, and $\overbar{EA}≅\overbar{OK}$.

Corresponding angles: $∠A≅∠K$, $∠B≅∠L$, $∠C≅∠M$, $∠D≅∠N$, and $∠E≅∠O$.

The corresponding parts depend upon the isometry used to establish the congruence.

In conclusion we can say that “corresponding parts of congruent polygons are congruent.” This statement is a consequence of the definition of congruence. It is often abbreviated “CPCTC.”

**Note to teachers:** Traditional geometry textbooks have had separate definitions for congruent segments, congruent angles, and congruent triangles. The transformational approach to congruence embedded in the Common Core standards has a single definition of congruence, based on rigid motions, that applies to all figures in the plane, including circles and polygons with any number of sides. Thus we suggest that you start the discussion above with polygons with more than three sides, even though we will be focusing mainly on pairs of congruent triangles in this course.

Introduce students to precise notation for indicating congruent polygons. It is critical that corresponding vertices be listed in the same order. In the case above we write

Pentagon *ABCDE*$≅$ Pentagon *KLMNO*. This is understood to indicate the following one-to-one correspondences: $A\leftrightarrow K$, $B\leftrightarrow L$, $C\leftrightarrow M$, $D\leftrightarrow N$, and $E\leftrightarrow O$.

**Differentiated Instruction (For Learners Needing More Help)**

If students are having difficulty identifying corresponding vertices, suggest that they place the name of one polygon underneath the other and line up the corresponding vertices as shown.



Students are now ready to apply the concept of corresponding parts. In **Activity 2.1.3** they are given pairs of congruent polygons and asked to identify corresponding sides and angles. In some of the exercises they can use the information given to find the measures of specified sides or angles. This activity introduces students to tick marks to indicate pairs of corresponding parts. You should explain this convention to students before they start the activity.

**Activity 2.1.4** **Area and Perimeter of Congruent Figures** is designed to help students discover another property of pairs of congruent figures: that they have equal areas and equal perimeters. This property applies to both polygons and circles. This activity provides an opportunity for students to recall the formulas for area, perimeter, and circumference that they learned in earlier grades.

**Group Activity 2.1.5** is based on the jigsaw method of group work. Assign students to groups of three. Each student is given one of three congruent triangles, the length of one side and the measure of one angle. Given the congruence relationship ∆MAP $≅∆FUR ≅∆HOT$, everyone in the group should be able to find the measures of all six parts of his or her triangle.

The final activity in this investigation, **Mapping Congruent Polygons**,has students find the rigid motions that map one of two congruent polygons onto the other. There are several versions of this activity. In **Activity 2.1.6a** the figures are placed in the coordinate plane and students apply the mapping rules developed in Unit 1. **Activity 2.1.6b** uses Geogebra and **Activity 2.1.6c** uses Geometer’s Sketchpad to achieve the same purpose.

At the end of the second day you may give **Exit Slip 2.1.2**, which asks students to find corresponding parts of congruent polygons.

**Journal Entry:** What property do pairs of congruent segments, congruent circles, and congruent triangles have in common? How can you tell if two figures are congruent? Look for students to describe congruence in terms of isometries.

**Closure Notes**

In this investigation we have discussed many different figures where the congruence relationship holds: segments, angles, circles, and polygons. Ask students what is the key idea that unifies all these relationships. Elicit from them the definition of congruence: Two figures are congruent if and only if one is the image of the other under a sequence of rigid motions (translations, rotations, and reflections). This concept is also addressed in the journal prompt. From this investigation students also learn that if two polygons are congruent, then the "corresponding parts" (sides, interior angles) are congruent.

**Vocabulary**

congruent

$≅$ (symbol for congruence)

corresponding parts

polygon

interior angle of a polygon

side of a polygon

**Resources and Materials**

Power point: Congruent segments and angles (to be developed)

Web sites on interchangeable parts:

<https://www.youtube.com/watch?v=sLb9QGWK8TA>.

<http://www.uh.edu/engines/epi1252.htm>.

Activities:

 Activity 2.1.1 Congruent Segments and Angles

 Activity 2.1.2 Properties of Congruence

 Activity 2.1.3 Congruent Polygons

 Activity 2.1.4 Areas and Perimeters of Congruent Figures

 Activity 2.1.5 Using Congruence to Find Unknown Parts (group activity)

 Activity 2.1.6 Mapping Congruent Polygons (three versions)

Rulers, protractors, graph paper for Activity 2.1.1 and Activity 2.1.6a

Geogebra for Activity 2.1.6b

Geometer’s Sketchpad for Activity 2.1.6c