**Unit 1: Investigation 7 (2 Days)**

**Isometries**

***Common Core State Standards***

* G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
* G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

**Overview**

This investigation reinforces the fact that isometries (translations, rotations, reflections, and glide reflections) preserve distance and angle. Students will compare and contrast this property of isometries with dilation (distance not preserved) and horizontal and vertical stretches (distance is not preserved and generally, neither are angles). Students will also see that we can take a figure and perform a series of isometries on it. This is in anticipation of the next unit where we will define congruent figures as ones where one can be mapped onto the other through a sequence of isometries.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Define the essential characteristic of an isometry
* Perform a dilation and a shear on a figure on the coordinate plane
* Perform a series of isometries on a given figure

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 1.7** asks students to define, in their own words, what an isometry is.
* **Journal Entry** asks students to talk about relationships they see between the pre-image and after image after performing a series of isometries on it.

**Launch Notes**

Begin the class with this video on geometric transformations used by Discovery Education when creating animated dinosaurs. It accomplishes a few goals. First, it serves as a nice review of reflections, translations, and rotations and reinforces the notion that these transformations preserve distance and angle size. Second, it shows a real-world use for them—and one that may interest and inspire some students. Be sure to point out that reflections are used on only part of the dragon—when one wing is mapped onto the other. Translations and rotations, on the other hand, are performed on the entire body of the dragon.

<https://www.youtube.com/watch?v=NY2cDTpsvBA>

**Teaching Strategies**

Up to this point in the unit, students have worked with four transformations – reflections, translations, rotations, and glide reflections.  **Activity 1.7.1** **Some New Transformations** is an examination of additional transformations such as dilations, stretches, and shears.

Students will first be instructed to perform a dilation on a figure. They have been exposed to dilations and scale factors in earlier grades so this should not be a completely foreign concept. Have them use the triangle with coordinates at (1,1), (5,1), and (2,3) and perform a dilation with a scale factor of 2 centered at the origin (Mapping rule (*x*, *y*) 🡪 (2*x*, 2*y*)). The activity will explain the steps necessary for performing this task. Have them measure each angle and side of the pre-image and the dilated image. They will eventually make connections between this and the previous four transformations. As a technology note, if trying to perform a dilation in Geogebra, ensure that you have first created a polygon and not simply created the points.

**Differentiated Instruction (For Learners Needing More Help)**

You may want to show the first 44 seconds of the following video to quickly explain what a dilation is: <https://www.youtube.com/watch?v=kX5VN85p5M4>

Additionally, students will also be asked to perform a stretch; for example, a horizontal stretch with rule (*x*, *y*)🡪 (3*x*, *y*) or a vertical stretch such as (*x*, *y*)🡪 (*x*, 3*y*). They will examine how these stretches impact measurements. Students will also be exposed to a shear such as
(*x*,*y*) 🡪 (*x* + *y*, *y*) that maps a rectangle onto an oblique parallelogram.

**Activity 1.7.2a Understanding Isometries** is an investigation into the differences between the four transformations that were discussed earlier in the unit (reflection, translation, rotation, glide reflection) and the ones addressed in the previous activity. Students should come away from this activity understanding that the first four are *isometries*—transformations that preserve size and angle, while the others do not preserve both. Additionally, although this activity is designed to be completed using graph paper, rulers, and protractors, it can also be done using Geogebra (**Activity 1.7.2b).**

Note: for **Activity 1.72a** graph paper with a one-centimeter or half-inch grid is preferred.

Have students construct an object and measure the length of its sides and its angles. They will then rotate, reflect, translate, and perform a glide reflection—each time measuring the sides and angles of the new image. Additionally, they will be writing down the coordinates of the pre-image as well as the transformed images.

**Differentiated Instruction (Enrichment)**

For most students, you may want to restrict the objects to triangles or parallelograms This limits the number of measurements they need to do. Conversely, you may challenge students looking for more enrichment to come up with more elaborate objects.

Students will compare these results with those from **Activity 1.7.1** to reinforce the notion that the isometries preserve size and angle measure.

Explain that the rotation, translation, reflection, and glide reflections are called “isometries.” An isometry is a transformation that preserves the exact size and shape of the object. The dilation, while preserving the angle measure, does not preserve the size of the object and is therefore, not an isometry. Similarly, the shear and stretch do not preserve either measurement and they are not isometries.

**Group Activity**

Activity 1.7.2a lends itself well to students working in groups of 4. Questions 7-9 are specifically designed for the jig-saw approach with each student becoming the “expert” on one of the four transformations. In addition you may assign to each student only one of the isometries in questions 2-5.

At the end of Day 1 you may give students **Exit Slip 1.7**, which asks students to define, in their own words, what an isometry is.

**Activity 1.7.3 Properties of Isometries** delves further into the concept of isometries by introducing four postulates that will be used throughout the course. Students will use Geogebra to discover the following postulates:

**Isometry Postulate** (applies to Translations, Rotations, Reflections and Glide Reflections): All isometries preserve distance and angle measure. Pairs of parallel lines are mapped onto parallel lines. Midpoints of segments are mapped onto midpoints.

**Translation Postulate**: Under a translation by the vector from *X* to *Y*, a line parallel to $\overleftrightarrow{XY}$ is mapped onto itself. A line that is not parallel to $\overleftrightarrow{XY}$ is mapped onto another line that is parallel to itself.

**Rotation Postulate**: Under a rotation about a point *P*, the point *P* is mapped onto itself. A line through point *P* is mapped onto another line through *P*. If the angle of rotation is 180° a line through *P* is mapped onto itself

**Reflection Postulate**: Under a reflection about line *l* every point on *l* is mapped onto itself. A line that is parallel to *l* is mapped onto another line that is parallel to *l*.

The Geogebra files ctcoregeomACT1731 and ctcoregeomACT1732 are used with this activity and can be found in the folder entitled Activity\_1\_7\_3\_gbb\_files as well as on GeogebraTube.

**Journal Prompt:** After you have performed an isometry on a figure, in what way are the pre-image and the image alike? Look for students to recognize that the distances and angle measures remain the same.

**Closure Notes**

In this investigation we have reviewed the four transformations that were examined previously and students learned that these four are called isometries because they preserve the size of the object and the measure of the angles. They were introduced to four postulates based on their observations of the effects of isometries. In comparison, students saw how dilations and stretches do not preserve both measurements. Additionally, students further explored the concept of composition by performing multiple isometries on a figure. This process is in anticipation of the formal definition of congruence, which will appear in the next unit.

**Vocabulary**

dilation

horizontal stretch

isometry

shear

vertical stretch

**Postulates and Theorems**

**Isometry Postulate** (applies to Translations, Rotations, Reflections and Glide Reflections): All isometries preserve distance and angle measure. Pairs of parallel lines are mapped onto parallel lines. Midpoints of segments are mapped onto midpoints.

**Translation Postulate**: Under a translation by the vector from *X* to *Y*, a line parallel to $\overleftrightarrow{XY}$ is mapped onto itself. A line that is not parallel to $\overleftrightarrow{XY}$ is mapped onto another line that is parallel to itself.

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**Reflection Postulate**: Under a reflection about line *l* every point on *l* is mapped onto itself. A line that is parallel to *l* is mapped onto another line that is parallel to *l*.

 **Resources and Materials**

Launch video: <https://www.youtube.com/watch?v=NY2cDTpsvBA>

Dilations video: <https://www.youtube.com/watch?v=kX5VN85p5M4> (stop at 0:44 seconds)

Activities:

Activity 1.7.1 Some New Transformations

Activity 1.7.2a Understanding Isometries (with Coordinates,)

Activity 1.7.2b Properties of Isometries (with Geogebra)

Activity 1.7.3 Properties of Isometries

Rulers, protractors, graph paper for Activity 1.7.1, 1.7.2a, and 1.7.3.

Geogebra for Activity 1.7.2b, and 1.7.3.