**Activity 2.6.2 – Solving Radical Equations**

Radical equations are equations that involve radical expressions. This activity focuses on solving different types of radical equations and using graphs to better understand radical equations.

1. Solve the following radical equations algebraically:
2. $\sqrt{2x-6}=8$
3. $5\sqrt{3x+2}=40$
4. $\sqrt{x-7}=7-\sqrt{x}$
5. $\sqrt{x+4}=\sqrt{2x-5}$

Sometimes when we square both sides of an equation a quadratic equation emerges. In some cases the quadratic equation that emerges has solutions that are not solutions of the original radical equation. These extra solutions are called *extraneous solutions*. To verify that a solution is not extraneous, we must check all solutions in the original equation.

Let’s use graphs to visualize the emergence of extraneous solutions. Consider the equation $\sqrt{2x+1}=x-7$. We will treat each side of the equation as a separate function:

$$y\_{1}=\sqrt{2x+1} y\_{2}=x-7$$

1. Graph the functions $y\_{1}$ and $y\_{2}$ on the coordinate plane below.

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1. Based on the graph, what is the solution of the equation $\sqrt{2x+1}=x-7$.
2. Let’s now explore the solutions from an algebraic perspective. To algebraically solve the equation $\sqrt{2x+1}=x-7$, first square both sides.
3. Now, rewrite your equation into a quadratic equation in standard form: $ax^{2}+bx+c=0$.
4. On the coordinate plane below, graph the quadratic function you obtained in Question 5.



1. Based on the graph of the quadratic function, what are the solutions of the quadratic equation you derived in Question 5.
2. Do the solutions from Question 7 satisfy the original equation? Why or why not?

**Extraneous Solutions**

You have just seen an equation with *extraneous solutions* – solutions that emerge in the equation-solving process but *fail* to satisfy the original equation. Extraneous solutions often emerge when we square both sides of an equation. Consider how the solution sets change when the following equations are transformed by squaring both sides.

1. $x=-5$
2. $\sqrt{x}=x-2$
3. Solve the following radical equations algebraically. Check for extraneous solutions.
4. $\sqrt{16-6x}=x$
5. $\sqrt{3x+2}=2x-5$
6. $x-3=\sqrt{3x+7}$
7. Solve the equations in Question 9 using the graphing calculator. For each equation, treat each side of the equation as a separate function, graph both functions simultaneously, and find the intersection point.
8. Find the intersection point of the following curves.



$$y=\sqrt{25-x^{2}}$$

$$y=\sqrt{2x}$$