

The Mathematics Progressions



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Mathematics: 3 shifts

1. **Focus:** Focus strongly where the standards focus
2. **Coherence:** Think across grades, and link to major topics
3. **Rigor:** In major topics, pursue **conceptual understanding**, procedural skill and **fluency**, and **application**



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There has been a lot of attention given to the three shifts inherent in the common core math. While all three are important, coherence is the result of the progressions that are contained within the standards. This will be our lens for today's session.

Why Coherence?

- Mathematics is not a list of disconnected topics, tricks, or mnemonics;
- Mathematics is a coherent body of knowledge made up of interconnected concepts.
- The standards are designed around coherent progressions from grade to grade
- Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years.

2014 Common Core State Standards Initiative



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Coherence Across and Within Grades

- It's about math making sense.
- The power and elegance of math comes out through carefully laid progressions and connections within grades.



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Coherence is about math making sense.

There are two types of coherence found in the math standards. One is the coherence of topics across grades and the other is the coherence within a grade.

Coherence across grades within the standards is in having students apply learning from a previous grade to learn a new topic.

Coherence within a grade comes from the reinforcement of the major topics by utilizing a supporting topic.

These two types of coherence result from the carefully laid structure of the standards.

The Structure of the Standards

- Coherence is an important design element of the standards.
- “The Standards are not so much built from topics as they are woven out of progressions.”

Structure is the Standards, Publishers’ Criteria for Mathematics, Appendix



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In the activity we are going to do later in the presentation, you will see how themes, topics, and language build across grades in the CCSSM. There are many meaningful progressions of mathematics in the CCS. Before we get into the standards themselves, we need to understand what is meant by learning progressions.

What are learning progressions?

- “descriptions of the successively more sophisticated ways of thinking about an idea that follow one another as students learn” (Wilson & Bertenthal, 2005)
- “a picture of the path students typically follow as they learn...a description of skills, understandings, and knowledge in the sequence in which they typically develop” (Masters & Forster, 1996)
- “a sequenced set of subskills and bodies of enabling knowledge that, it is believed, students must master en route to mastering a more remote curricular aim” (Popham, 2008)
- “a description of how student understanding or learning can or should develop over time” (Gong, 2008)



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Description of successively more sophisticated ways of thinking about a big idea

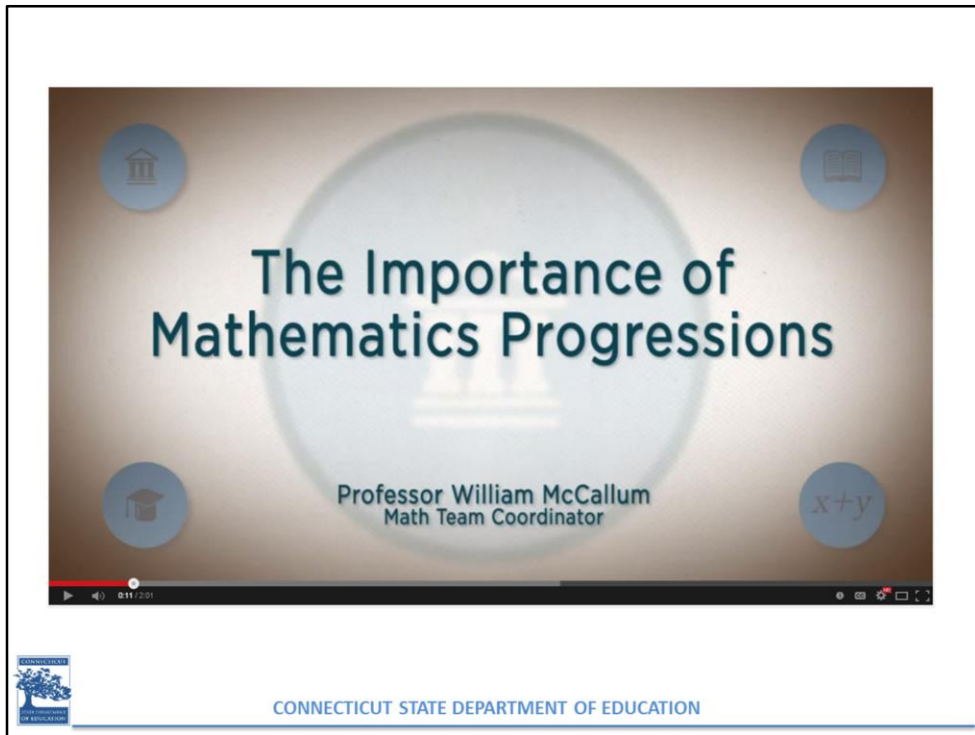
Provide a framework for long-term development

Describes what it means to move towards more expert understanding in an area

Gauge increasing competence over time

A sequence of successively more complex ways of thinking about how an idea develops over time

Consider how ideas build upon each other to form more complex practices or ideas



Let's listen to Bill McCallum as he explains progressions specifically around mathematics and the development of the Common Core.

<https://www.youtube.com/watch?v=a-P9KQdhE0U>

Mathematics Progressions: The beginning of the standards

- Narrative documents describing the progression of a topic across a number of grade levels
- Informed both by educational research and the structure of mathematics
- Sliced into grade level standards

The Common Core Standards Writing Team



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To summarize what we just heard Bill explain. Read to the slide.

Domains and Conceptual Categories

K-12

Common Core State Standards – Mathematics

Learning Progressions

Kindergarten	1	2	3	4	5	6	7	8	HS	
Counting and Cardinality									Number and Quantity	
Number and Operations in Base Ten					Ratios and Proportional Relationships					
			Number and Operations - Fractions			The Number System				
Operations and Algebraic Thinking						Expressions and Equations			Algebra	
							Functions		Functions	
Geometry						Geometry				Geometry
Measurement and Data						Statistics and Probability			Statistics and Probability	



Ohio Department of Education (12/14/10)

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Looking at the standards, this shows a graphic of the progressions of mathematics K-12. Note that in grades K-8 the mathematics is broken down by domain. This does not mean that the progression occurs only within the domain. There are also connections between domains. When students reach high school, the standards are broken down into conceptual categories. As this graphic shows, these conceptual categories tie together all of the domains that eh students learned throughout their K-8 careers.

A Closer Look into the Domains....

K.OA.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

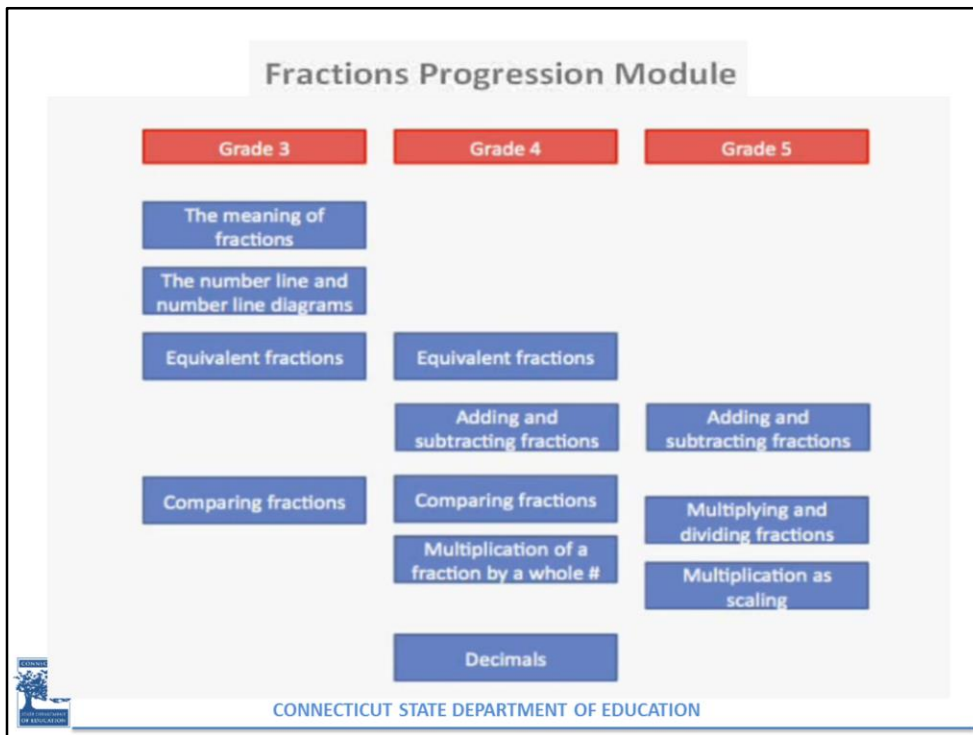
1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

2.OA.2: Fluently add and subtract within 20 using mental strategies. (Note: See standard 1.OA.6 for a list of mental strategies). By end of Grade 2, know from memory all sums of two one-digit numbers.



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To demonstrate an example of where we see a progression, we can look at the K-2 domain of Operations and Algebraic Thinking. You can see how the mathematics gets more complex as a student moves on. Talk to the bold part of the standard.



Now that we have a sense of the what we mean by progressions and understand their importance in the CCS math, an important question that needs to be examined is what does this mean for instruction and student learning. To help explain this we are going to look at a brief video clip on fractions. You can see from this graphic how the concept of fractions progresses from third grade to fifth grade. In the video we will see how this progression is used to explain the new concepts.
<https://www.youtube.com/watch?v=OZblmRwktTo>

A Closer Look into the Domains....

3.NF.3:

d. Compare two fractions with the same numerator or the same denominator

by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole.

Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

4.NF.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$.

Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

5.NF.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions

in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*

5



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We just saw a video that explains the progressions of fractions. It is important to always bring our instruction back to the standards. This example of standards for fractions from grades 3 to 5 demonstrates how students are taking their prior learning and building upon it as they advance through the grades.



Activity 1

Deep Dive into the Math



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I have done a lot of talking and now I want to give you the opportunity to explore progressions on your own. Let's start with the envelope labeled Activity 1. In it you will find three standards. I would like for you to arrange these standards in a progression and identify what grade level each would be expected.

Coherence in the Middle Grades

6.EE.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

7.EE.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. **b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers.** Graph the solution set of the inequality and interpret it in the context of the problem.

8.EE.8: Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b. Solve systems of two linear equations algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. **c. Solve real-world and mathematical problems leading to two linear equations in two variables.**



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Let's see how you did. These standards are from the Expressions and Equations domain for grade 6 to 8.

A Closer Look into the Conceptual Categories... Functions

Algebra 1	Geometry	Algebra 2
A.REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	G.GPE.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.



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Much of the discussion about progressions is from grades K-8. However, the progressions continue in the standards for high school. In Connecticut, most of the districts use the traditional pathway approach to mathematics. This shows that even in this pathway, there is evidence of a progression and that the progression does not need to be contained to one conceptual category.

Diving Deeper....

- Place the standards of each color under the appropriate grade (K-8).
- No grade has two of the same color card.
- Some colors that have only a few cards might represent consecutive grades and some may not.
- Read each card in it's entirety to help determine placement.
- Do not check your Standards until you and your colleagues agree on the final product.
- Discuss horizontal and vertical observations with your partners.



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We have seen examples of the progressions throughout the K-2, 3-5, 6-8 and high school grade bands. In order to really understand the power of the progressions and their impact on the mathematics you need to dig into multiple standards across a variety of grades. In this activity you will have the opportunity to do that. In your envelopes you will find several standards that are color coded. Your task is to arrange each color by placing it in the appropriate grade on the poster paper. There is only one color card for each grade and not all grades will have all colors. The colors that have fewer cards may or may not be consecutive grades.

The key is to read the standards on the cards. This isn't an activity to just see if you know the standards for each grade or can reason the order they should go in. Really read the standards. See the coherence of the writing. See how the topics progress from one grade to another. See why it is important for teachers to know the standards for their grade as well as for the grades before and after theirs.

This activity is most beneficial when not using the standards as a guide. We want you to have to mull over these standards and hash out tough decisions with your colleagues. When you are finished and comfortable with your choices, use the Standards to check your work.

This activity was adapted from Achieve. <http://achievethecore.org/page/400/deep-dive-into-the-math-shifts-list-pg>

Discussion

For who are the progressions for the CCS Math useful and in what ways can they be used effectively?



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Solicit responses from the participants.

Benefits of Using the Progressions

With a possible learning path in mind educators can...

- Consider strategies for instructional scaffolding to get students to the next stage of learning
- Use formative & summative assessments “strategically” and more frequently; they value “uncovering student thinking”
- Collaboratively analyze student work creating a deeper understanding of how learning develops
- Adjust instruction according to what students CAN do, not what they CANNOT do
- Shift perceptions, especially of their lower performing students & what to do next to support learning



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Read to the slide

Resources for Progressions

- <http://ime.math.arizona.edu/progressions/>
 - Updated versions of the early progressions drafts, revised and edited to correspond with the Standards
- http://ctcorestandards.org/?page_id=2
 - Compilation of the progression of standards by domain for K-2, 3-5 and 6-8
 - Compilation of the progression of standards by conceptual category for the traditional pathway approach to high school mathematics



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The progression documents that Bill McCallum discussed at the beginning of the presentation can be found through the University of Arizona website. These documents go into great detail about the progressions for each domain and the conceptual categories. These are rather lengthy so to look at the progression of the standards by grade band in an easier to use format, Dewey Gottlieb from Hawaii compiled the standards by domain for the K-2, 3-5, and 6-8 grade bands. Recently, I compiled the standards by conceptual category for the traditional pathway at the high school level. The standards at each grade level align with the model curriculum for Algebra 1 that is already available and will align to Geometry and Algebra 2 when it is released this summer.