

**Table 4: Key CCSS for Mathematical Practice<sup>25</sup> with Embedded Analytical Tasks and Receptive and Productive Language Functions**

This table defines in detail these practices by outlining the language functions that ELLs need to engage with mathematics content.

Mathematics Sense-Making and Language Use	
Key CCSS for Mathematical Practice 1: Make sense of problems and persevere in solving them	
Analytical Tasks	<ul style="list-style-type: none"> <li>• Explain to self a problem’s meaning, look for entry points to solution, and plan solution pathway</li> <li>• Analyze givens, constraints, relationships, and goals</li> <li>• Make conjectures about form and meaning of solution</li> <li>• Consider analogous problems</li> <li>• Monitor effectiveness of current selected solution strategy and decide when to pursue a different solution strategy</li> <li>• Check answers using different methods</li> <li>• Understand others’ approaches to solving complex problems and identify correspondences between them</li> <li>• Create coherent representation of problems, considering units</li> <li>• Monitor use of resources such as time and effectiveness of current selected solution strategy</li> <li>• Monitor and evaluate reasonableness of intermediate and final results</li> </ul>
Receptive Language Functions	<ul style="list-style-type: none"> <li>• Comprehend the meaning of a problem as presented in multiple representations, such as spoken language, written texts, diagrams, drawings, tables, graphs, and mathematical expressions or equations</li> <li>• Comprehend others’ talk about math problems, solutions, approaches, and reasoning</li> <li>• Coordinate texts and multiple representations</li> </ul>
Productive Language Functions	<p>Communicate (orally, in writing, and through other representations) about concepts, procedures, strategies, claims, arguments, and other information related to problem solving:</p> <ul style="list-style-type: none"> <li>• Create, label, describe, and use in presenting solutions to a math problem multiple written representations of a problem<sup>26</sup></li> <li>• Explain in words orally or in writing relationships between quantities and multiple representations of problem solutions</li> <li>• Present information, description of solutions, explanations, and arguments to others</li> <li>• Respond to questions or critiques from others</li> <li>• Ask questions about others’ solutions, strategies, and procedures for solving problems</li> </ul>

<sup>25</sup> These practices are intended to apply systematically across grade level content in mathematics; they are not free-floating, but well-grounded in the content standards. Neither are the practices static across grades but instead should be tailored to the content of the grade and to grade-level appropriate student thinking.

<sup>26</sup> Multiple representations include written text, diagrams, drawings, symbols, mathematical expressions or equations, tables, graphs, mathematical models, and/or pictures of math manipulatives or other objects.

Key CCSS for Mathematical Practice 2: Reason abstractly and quantitatively	
Analytical Tasks	<ul style="list-style-type: none"> <li>• Know when it is best to abstract a given problem situation, represent it symbolically, and manipulate symbols without necessarily attending to referents (decontextualize)</li> <li>• Know when it is best to pause as needed during symbol manipulation to use the meaning of the symbols involved (contextualize)</li> <li>• Monitor and decide when to contextualize and decontextualize</li> <li>• Attend to meaning of quantities in the problem situation</li> <li>• Do and undo computations; abstract from computation</li> </ul>
Receptive Language Functions	<ul style="list-style-type: none"> <li>• Comprehend the meaning of a problem situation and its relevant quantities as presented through multiple representations</li> <li>• Comprehend others' talk about the relevant and irrelevant quantities in the problem situation</li> <li>• Coordinate written texts and multiple representations</li> </ul>
Productive Language Functions	<p>Communicate (orally, in writing, and through other representations) about concepts, procedures, strategies, claims, arguments, and other information related to abstract and quantitative reasoning:</p> <ul style="list-style-type: none"> <li>• Explain reasoning as it relates to problem situation, especially the quantities in the problem that are mathematically relevant</li> <li>• Create, label, describe, and defend coherent representations of the problem situation at hand</li> <li>• Ask questions to contextualize the problem situation or the quantities in the problem</li> </ul>
Key CCSS for Mathematical Practice 3: Construct viable arguments and critique the reasoning of others	
Analytical Tasks	<ul style="list-style-type: none"> <li>• Understand and use stated assumptions, definitions, and previously established results</li> <li>• Make conjectures and build logical progression of statements to explore truth of conjectures</li> <li>• Justify conclusions, communicate them to others, and respond to counterarguments</li> <li>• Analyze situations by breaking them into cases</li> <li>• Recognize and use counterexamples</li> <li>• Make plausible arguments taking into account context from which data arose</li> <li>• Compare effectiveness of two plausible arguments</li> <li>• Identify correct vs. flawed logic/reasoning</li> <li>• Monitor one's own and others' reasoning</li> </ul>
Receptive Language Functions	<p>Comprehend oral and written concepts, procedures, or strategies used in arguments and reasoning, including</p> <ul style="list-style-type: none"> <li>• Questions and critiques using words or other representations</li> <li>• Explanations offered using words or other representations by others (peers or teachers)</li> <li>• Explanations offered by written texts using words or other representations</li> </ul>

<p>Productive Language Functions</p>	<p>Communicate using words (orally and in writing) about concepts, procedures, strategies, claims, arguments, and other information related to constructing arguments and critique reasoning:</p> <ul style="list-style-type: none"> <li>• Provide written or verbal explanation of an argument using words through logical progression of statements, and also using multiple non-verbal representations, concrete referents (such as objects), or more formal means (i.e., mathematical symbols and mathematical proofs)</li> <li>• Justify conclusions and respond to counterarguments</li> <li>• Recognize and use counterexamples</li> <li>• Respond to questions by amplifying explanation</li> <li>• Respond to critiques by countering with further explanation or by accepting as needing further thought</li> <li>• Critique or support explanations or designs offered by others</li> </ul>
<p style="text-align: center;"><b>Key CCSS for Mathematical Practice 4: Model with mathematics</b></p>	
<p>Analytical Tasks</p>	<ul style="list-style-type: none"> <li>• Apply math to everyday situations (e.g., outside of school and on the job)</li> <li>• Pose a problem for a situation that can be solved with the available data and by using mathematical models</li> <li>• Make assumptions and approximations to temporarily simplify a complicated problem situation</li> <li>• Identify and map relationships among important quantities; decide which quantities are relevant</li> <li>• Analyze relationships among quantities mathematically to draw conclusions</li> <li>• Interpret results in context of the situation</li> <li>• Monitor one’s own and others’ reasoning in support of a model</li> <li>• Reflect on reasonableness of results and improve model as needed</li> <li>• Use technology to visualize results, explore consequences, and compare predictions with data</li> </ul>
<p>Receptive Language Functions</p>	<ul style="list-style-type: none"> <li>• Comprehend others’ oral or written descriptions, defenses, and discussions of their models</li> <li>• Comprehend the meaning of models presented in multiple representations</li> </ul>
<p>Productive Language Functions</p>	<p>Communicate (orally and in writing) about concepts, procedures, strategies, claims, arguments, and other information related to mathematical models:</p> <ul style="list-style-type: none"> <li>• Label (or create and label) diagrams of a model</li> <li>• Describe and defend a model using words and other representations</li> <li>• Ask questions and hypothesize about whether or how others’ models work</li> </ul>
<p style="text-align: center;"><b>Key CCSS for Mathematical Practice 5: Use appropriate tools<sup>27</sup> strategically</b></p>	
<p>Analytical Tasks</p>	<ul style="list-style-type: none"> <li>• Make sound decisions about helpfulness of different tools for problem solving</li> <li>• Use estimation and other strategies to detect possible errors in computation</li> <li>• Use technology to explore and deepen conceptual understanding, visualize results, explore consequences, and compare predictions with data</li> <li>• Identify and use relevant mathematical resources such as digital content on websites</li> </ul>

<sup>27</sup> Tools include algorithms (e.g., the instructed procedure for double digit multiplication), strategies (e.g., estimation), technology (e.g., calculators and websites), and visual media (e.g., dynamic models and simulations).

<p>Receptive Language Functions</p>	<ul style="list-style-type: none"> <li>• Comprehend others’ oral and written language that describes purposes and functions of tools and other resources</li> <li>• Comprehend the purposes and functions of tools and other resources as presented in texts, diagrams, and visual media</li> </ul>
<p>Productive Language Functions</p>	<p>Communicate (orally and in writing) about concepts, procedures, strategies, claims, arguments, and other information related to strategic use of tools:</p> <ul style="list-style-type: none"> <li>• Ask questions regarding purpose and functions of tools and others’ use of them</li> <li>• Explain own use of tools and outcomes of tool use</li> </ul>
<p><b>Key CCSS for Mathematical Practice 6: Attend to precision</b></p>	
<p>Analytical Tasks</p>	<ul style="list-style-type: none"> <li>• When appropriate, communicate precisely with others about mathematical reasoning and objects (e.g., use clear definitions of terms, state meaning of symbols used, specify units of measure, label visual representations, and make claims that apply to a precise set of situations)</li> <li>• Refine communication about mathematical reasoning and objects so that it increasingly becomes more mathematically precise (e.g., uses clearer definitions of terms, explicitly states the meaning of symbols used, specifies units of measure)</li> <li>• Calculate, compute, and use arithmetic procedures appropriately, accurately, and efficiently</li> <li>• Express numerical answers with degrees of precision appropriate for the problem situation</li> <li>• Monitor one’s own and others’ use of precision</li> <li>• Decide when precision is more necessary (e.g., during a presentation) and when it is not a high priority (e.g., during exploration and exploratory talk in groups)</li> <li>• Decide the level of precision necessary (e.g., one can make a precise claim that only applies to a defined set of instances even when using colloquial or imprecise individual words).</li> </ul>
<p>Receptive Language Functions</p>	<ul style="list-style-type: none"> <li>• Comprehend others’ spoken language regarding definitions, meaning of symbols, arithmetic procedures, strategies, solutions, claims, evidence, etc.</li> <li>• Comprehend the meaning and features of precision of definitions, symbols meanings, units of measure, and visual representations as presented in multiple representations (e.g., texts, diagrams, and visual media)</li> </ul>
<p>Productive Language Functions</p>	<p>Communicate with precision (orally, in writing, and through other representations) about claims and arguments related to precision:</p> <ul style="list-style-type: none"> <li>• Define key terms and concepts</li> <li>• Explain meaning of symbols</li> <li>• Specify units of measure</li> <li>• Label (or create and label) visual representations</li> <li>• Ask questions to clarify meaning of others’ statements or representations</li> <li>• Make specific claims and evaluate constraints</li> </ul>
<p><b>Key CCSS for Mathematical Practice 7: Look for and make use of structure</b></p>	
<p>Analytical Tasks</p>	<ul style="list-style-type: none"> <li>• Look closely to discern pattern or structure (e.g., look for patterns in quantities, relationships among quantities, arithmetic procedures, data in tables, and graphs)</li> <li>• Shift perspective on a problem situation or a mathematical representation (e.g., equation, table, or graph) if necessary</li> <li>• See complicated mathematical representations, such as algebraic expressions, equations, or lines, as a process, single objects, or as composed of several objects</li> <li>• Flexibly use different perspectives of mathematical representations</li> <li>• Monitor and decide which perspective is most useful for the problem situation at hand</li> </ul>

Receptive Language Functions	<ul style="list-style-type: none"> <li>• Comprehend the meaning of patterns or structures found in a situation, problem, or mathematical expression as presented in spoken language, texts, and diagrams</li> <li>• Comprehend others' talk about patterns and structures</li> </ul>
Productive Language Functions	<p>Communicate (orally, in writing, and through other representations) about concepts, procedures, strategies, claims, arguments, and other information related to structure:</p> <ul style="list-style-type: none"> <li>• Create and label representations of patterns or structures</li> <li>• Describe patterns or structures</li> <li>• Ask questions about others' use of patterns or structures</li> </ul>
<b>Key CCSS for Mathematical Practice 8: Look for and express regularity in repeated reasoning</b>	
Analytical Tasks	<ul style="list-style-type: none"> <li>• Notice if calculations are repeated (i.e., reflect on arithmetic procedures)</li> <li>• Look both for general methods or solution strategies (generalize) and for shortcuts</li> <li>• Monitor reasoning process while attending to detail</li> <li>• Monitor and evaluate reasonableness of intermediate and final results</li> <li>• Search for regularity or trends in multiple representations (e.g., look for regularity in relationships among quantities, data in tables, and graphs)</li> <li>• Graph data and search for regularity or trends</li> <li>• Abstract from computation, build rules to represent functions</li> </ul>
Receptive Language Functions	<ul style="list-style-type: none"> <li>• Comprehend others' oral and written language and other representations regarding regularity (e.g., repetition of calculations, methods used, or evaluation of intermediate and final results)</li> <li>• Comprehend descriptions, discussions, and arguments about regularity (i.e., repeated patterns, discussions of methods or solution strategies, or evaluations of intermediate results as presented in multiple representations)</li> </ul>
Productive Language Functions	<p>Communicate (orally, in writing, and through other representations) about concepts, procedures, strategies, claims, arguments, and other information related to regularity in repeated reasoning:</p> <ul style="list-style-type: none"> <li>• Ask questions about others' use of repetition, methods or solution strategies, and evaluation of intermediate and final results</li> <li>• Explain patterns, discuss methods or solution strategies, and evaluations of results</li> </ul>

Council of Chief State School Officers. (2012). Framework for English Language Proficiency Development Standards corresponding to the Common Core State Standards and the Next Generation Science Standards. Washington, DC: CCSSO. Used with permission.

**ANALYTICAL  
TASKS  
(3)**

**RECEPTIVE  
LANGUAGE  
FUNCTIONS  
(3)**

**PRODUCTIVE  
LANGUAGE  
FUNCTIONS  
(3)**

**Explain to self a  
problem's meaning,  
look for entry points  
to solution, and plan  
solution pathway**

**Analyze givens,  
constraints,  
relationships,  
and goals**

**Make conjectures  
about form and  
meaning of solution**

**Comprehend the  
meaning of a problem  
as presented in multiple  
representations** such as  
spoken language, written texts,  
diagrams, drawings, tables, graphs,  
and mathematical expressions or  
equations

**Comprehend  
others' talk about  
math problems,  
solutions, approaches  
and reasoning**

**Coordinate texts  
and multiple  
representations**

**Create, label,  
describe, and use in  
presenting solutions  
to a math problem  
multiple written  
representations  
of a problem**

**Explain in words  
orally or in writing  
relationships between  
quantities and multiple  
representations of  
problem solutions**

**Present information,  
description of  
solutions, explanations,  
and arguments to  
others**



**ANALYTICAL  
TASKS  
(3)**

**RECEPTIVE  
LANGUAGE  
FUNCTIONS  
(2)**

**PRODUCTIVE  
LANGUAGE  
FUNCTIONS  
(3)**

**When appropriate, communicate precisely with others about mathematical reasoning and objects** (e.g., use clear definitions of terms, state meaning of symbols used, specify units of measure, label visual representations, and make claims that apply to a precise set of situations)

**Refine communication about mathematical reasoning and objects so that it increasingly becomes more mathematically precise**  
(e.g., uses clearer definitions of terms, explicitly states the meaning of symbols used, specific units of measure)

**Calculate, compute and use arithmetic procedures appropriately, accurately, and efficiently**

**Comprehend others' spoken language regarding definitions, meaning of symbols, arithmetic procedures, strategies, solutions, claims, evidence, etc.**

**Comprehend the meaning and features of precision of definitions, symbols, meanings, units of measure, and visual representations as presented in multiple representations (e.g., text, diagrams, and visual media)**

**Define key terms  
and concepts**

**Explain meaning  
of symbols**

**Specify units  
of measure**



# Making Matchsticks

Level: Middle School

Version: 7.2.13

This task is intended to help teachers assess how well students are able to:

- Interpret a situation and represent the variables mathematically.
- Select appropriate mathematical methods.
- Interpret and evaluate the data generated.
- Communicate their reasoning clearly.

It is accompanied by a formative assessment lesson.

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**Language of Mathematics Task: Mathematically Speaking** 12

This Language of Mathematics task was designed to support ELL students in learning to talk about the mathematics in Making Matchsticks. It is accompanied by suggestions for classroom use.

Name \_\_\_\_\_

## Making Matchsticks

Matchsticks are rectangular prisms of wood measuring approximately:

$$\frac{1}{10} \text{ inch by } \frac{1}{10} \text{ inch by } 2 \text{ inches}$$

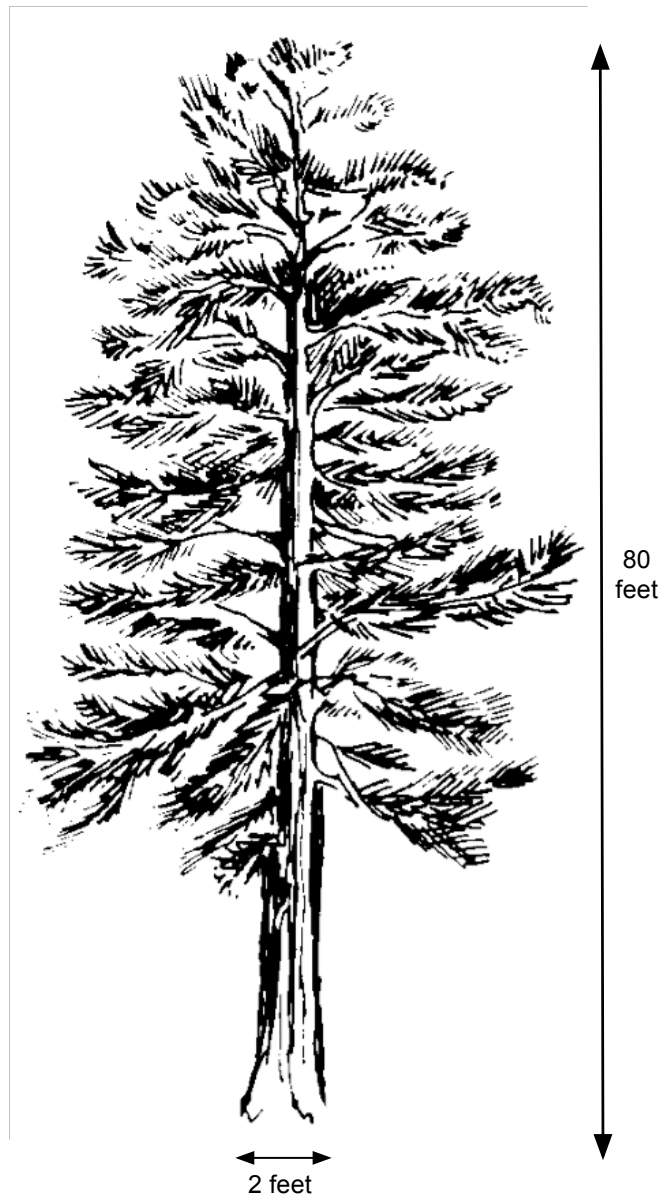


Matchsticks are often made from pine trees.

Estimate how many matchsticks can be made from this tree.

You may find some of the information given on the formula sheet helpful.

Explain your work carefully, giving reasons for any choices you make.



Student Materials

Modeling: *Making Matchsticks*  
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## MAKING MATCHSTICKS | ANNOTATIONS

### Core Idea

This task and the associated formative assessment lesson afford opportunities for students to develop strategies for solving problems. Students select formulas from a sheet of geometric formulas in order to model a situation. They interpret given data, make approximations, and communicate their reasoning in verbal and written form. Students analyze and critique solutions developed by others.

Note that although the task was designed to have students select volume formulas from a list, students are expected to know these formulas by the end of grade 8. This task draws on understandings of rate and proportional reasoning (a focus of grades 6 and 7), geometric measurement and volume which begins with right rectangular prisms in grade 5, and is extended in grades 6, 7, and 8. It is an opportunity to build toward the high school number and quantity standard of interpreting units consistently in formulas.

### Common Core State Standards for Mathematics

<http://www.corestandards.org/Math/Content/8/G>

*Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.*

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

### Common Core State Standards for Mathematical Practice

<http://www.corestandards.org/Math/Practice>

. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. . . . They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. . . .

Grade 8,  
Geometry (p. 56)

**SMP.1. Make sense of problems and persevere in solving them.**

**SMP.3. Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. . . .

**SMP.4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. . . . They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.

**SMP.6. Attend to precision.**

Mathematically proficient students try to communicate precisely to others. . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . .

**Common Core State Standards for ELA/Literacy**

<http://www.corestandards.org/ELA-Literacy>

**Grade 6, Writing, Text Types and Purposes (p. 42)**

1. Write arguments to support claims with clear reasons and relevant evidence.

**Grade 6, Speaking & Listening, Comprehension and Collaboration (p. 49)**

1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

**Grade 6, Speaking & Listening, Comprehension and Collaboration (p. 49)**

2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how it contributes to a topic, text, or issue under study.

**Grade 6, Speaking & Listening, Presentation of Knowledge and Ideas (p. 49)**

5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information.



**Grade 6, Reading  
Standards for  
Informational Text,  
Integration of  
Knowledge and  
Ideas (p. 39)**

7. Integrate information presented in different media or formats (e.g., visually, quantitatively) as well as in words to develop a coherent understanding of a topic or issue.

## Comments

*Purpose of the task.* Making Matchsticks is part of a MARS Formative Assessment Lesson (FAL), which can be downloaded here: <http://map.mathshell.org/materials/lessons.php?gradeid=23>.

In these FALs, initial individual student work on the task should not be supported or scaffolded by the teacher because their work is intended to produce evidence of their current knowledge. This assessment of students' prior knowledge will be used as the basis for the lesson.

*Using the task with ELLs.* Although students' mathematical work should not be supported or scaffolded, teachers will gain significantly more information about students' prior knowledge of the mathematics needed for the task if students comprehend what they are being asked to do in the task. In other words, although an unsupported attempt at the task by an ELL student will generate evidence of how well the student understood the task instructions, the attempt is less likely to inform a teacher about the mathematics the student would bring to the task with a clear understanding of what was being asked. Because of this, even if the task is to be used for individual assessment, we recommend that the teacher provide some support for ELLs for the individual work on the initial task for MARS Formative Assessment Lessons. The sole purpose of this support should be to give students access to the task—not directly to the mathematics they need to do the task, but rather to a clear understanding of what they are being asked to do.

*Outline of the lesson.* The MARS student materials include the task, a formula sheet, and a questionnaire called How Did You Work? After students have worked on the task individually (in class or as homework), they discuss the task and various responses (as described in detail in the MARS materials). Each small group of students shares a large sheet of paper for making a poster and copies of sample responses.

Here is the outline as given in the MARS materials:

1. Before the lesson, students tackle the problem individually. You assess their responses and formulate questions that will prompt them to review their work.
2. At the start of the lesson, students respond individually to the questions set, and then work in groups to combine their thinking and produce a collaborative solution in the form of a poster.

3. In the same small groups, students evaluate and comment on some sample responses. They identify the strengths and mistakes in these responses and compare them with their own work.
4. In a whole-class discussion, students explain and compare the solution strategies they have seen and used.
5. Finally, students reflect on their work and their learning.

The suggestions and the Mathematically Speaking task in these annotations are intended to supplement the lesson described in this outline. The suggestions supplement parts 1, 2, 3, and 5. The Mathematically Speaking task supplements the poster presentations in part 4.

Some potential challenges in the task are that students might:

- not take into consideration the different units used for toothpick and tree measurements.
- not understand and apply the various formulas for volume.
- misplace the decimal.
- envision the tree as two-dimensional.

For more potential challenges, see the table of possible responses in the next section.

### **Suggestions**

The minimal support we recommend for using Making Matchsticks as an individual assessment is to ask students to mark the task, replacing “inch” by “in” and “80 feet” by “80 ft,” and to identify the objects on the upper right as matchsticks. This support is intended only to present the question and the given information more clearly, and not to add information or suggest a solution path.

*Before students work on Making Matchsticks.* Instructions from the teacher should be clear, direct, and concise. The direction in this activity might be reduced to the following:

- Estimate how many matches can be made from the wood in this tree.
- Use the relevant information on the formula sheet. It will help you find some answers.

- Read the task, and show all your work. Showing your work helps me understand your reasoning (thinking).
- It is important that your work is organized and presented in a clear manner (way).

*Before the lesson: Assessing students' responses on Making Matchsticks.* The Common Issues table below is copied and pasted from the online teacher materials for the MARS lesson Making Matchsticks; additional suggestions and prompts from the Understanding Language Project are given below this table.

Common issues:	Suggested questions and prompts:
<p><b>Student has difficulty getting started</b></p>	<ul style="list-style-type: none"> <li>• What do you know? What do you need to find out?</li> <li>• How could you simplify the problem?</li> </ul>
<p><b>Student ignores the units</b> For example: The student calculates the volume of a matchstick in cubic inches and the volume of the tree trunk in cubic feet.</p>	<ul style="list-style-type: none"> <li>• What measurements are given?</li> <li>• Does your answer seem reasonable if you consider the size of a matchstick compared to the size of a pine tree?</li> </ul>
<p><b>Students makes incorrect assumptions</b> For example: The student multiplies the volume of the tree trunk in cubic feet by 12 and assumes this gives the volume of the tree trunk in cubic inches.</p>	<ul style="list-style-type: none"> <li>• Can you explain why you have multiplied by 12?</li> <li>• When you figure out a volume how many dimensions do you multiply together? How does this calculation effect how you convert the volume from cubic feet to cubic inches?</li> <li>• Can you describe the dimensions of the tree in inches? What do you notice?</li> </ul>
<p><b>Student uses an inappropriate formula</b> For example: The student calculates the <i>surface area</i> of a rectangular prism from the dimensions given for the tree.</p>	<ul style="list-style-type: none"> <li>• Does your choice of formula make good use of all the wood in the tree trunk?</li> <li>• Is this the best model for a tree trunk?</li> <li>• What is the difference between area and volume?</li> </ul>
<p><b>Students' work is unsystematic</b></p>	<ul style="list-style-type: none"> <li>• Would someone in your class who has not used this method be able to follow your work?</li> <li>• Can you describe your method as a series of logical steps?</li> </ul>
<p><b>Students' work is poorly presented</b> For example: The student underlines numbers and it is left to the reader to work out why this is the answer as opposed to any other calculation.</p>	<ul style="list-style-type: none"> <li>• Can you explain each part of your solution?</li> <li>• What does each of these calculations represent?</li> <li>• Can you justify the choices you have made?</li> </ul>
<p><b>Student has difficulties when substituting into a formula</b> For example: The student multiplies the radius by 2, rather than squaring, when using the formula for the volume of a cone/cylinder. Or: The student substitutes diameter rather than radius into the formula for the volume of a cone/cylinder.</p>	<ul style="list-style-type: none"> <li>• What is the difference in meaning between <math>2r</math> and <math>r^2</math>?</li> <li>• Does your answer seem reasonable?</li> <li>• How can you check your work against the information given in the problem?</li> </ul>

<p><b>Students' work is incomplete</b></p> <p>For example: The student does not divide the volume of the tree trunk by the volume of a matchstick.</p>	<ul style="list-style-type: none"> <li>• What do your calculations represent?</li> <li>• Have you found out how many matchsticks can be made from the tree?</li> </ul>
<p><b>Student rounds to one or more decimal places</b></p>	<ul style="list-style-type: none"> <li>• Why won't part of a matchstick count in your estimate?</li> </ul>
<p><b>Student completes the task</b></p>	<ul style="list-style-type: none"> <li>• How can you check that the method you have used has given a reasonable estimate?</li> <li>• Can you try a different method to check your answer?</li> <li>• What assumptions have you made?</li> </ul>

Below are additional issues not found in the existing Common Issues table for this lesson, together with suggested questions and prompts. Note that these provide considerably more scaffolding than the prompts above.

<p><b>Student attempts to use proportional reasoning to develop a solution path but gets confused.</b></p>	<ul style="list-style-type: none"> <li>• What is the rate of inches per foot?</li> <li>• What is the rate of cubic inches per cubic foot?</li> <li>• How can you use these rates to explain your solution to the problem?</li> </ul>
<p><b>Student struggles to make sense of linear, square, and cubic units.</b></p>	<ul style="list-style-type: none"> <li>• Draw three figures: first, draw a length of 1 inch; second draw a square with 1 inch side lengths; third, draw a cube with 1 inch side lengths. Then carefully label the side lengths in each drawing.</li> <li>• (If a student does not understand the exercise above, model each drawing with 1 inch, then ask the student to do the exercise with 1 foot. Then ask how many linear inches are in each linear foot, how many square inches are in each square foot, and how many cubic inches are in each cubic foot.)</li> </ul>
<p><b>Student has trouble getting started.</b> (Note: this is the entry in the first row of the existing table; to the right is an additional prompt for this entry.)</p>	<ul style="list-style-type: none"> <li>• Re-read the problem aloud to your partner(s).</li> </ul>
<p><b>Student ignores the units.</b> (Note: This is the entry in the second row of the existing table; to the right are an additional prompt and question that belong between the two existing questions.)</p>	<ul style="list-style-type: none"> <li>• What is being measured in this problem?</li> <li>• Try to explain to your partner(s) how the measurements you are given can help you find the measurements you need in order to solve the problem.</li> </ul>

*Beginning of the lesson: Reviewing individual solutions.* Include a printed list of questions on which students can reply.

Some possible sentence starters for review process:

- At first I thought . . .
- I realized that . . .
- However . . .

- Now, I see that . . .
- Next time, I will . . .

*Whole class discussion after students have discussed sample approaches.* Possible questions to ask:

- Which approach or paper made the most sense to you? (Like and dislike may not be the best language to use here.) In which way(s)?
- Which approach was difficult to understand? Why?

*At the end of the lesson or for homework.* Possible questions to ask:

- Think about the different methods we used in this lesson.
- What was the most important thing you learned from your work today?
- Write about one thing that you are wondering about, find unclear, or need help with.
- Share your writing with another student.

## LANGUAGE OF MATHEMATICS TASK MATHEMATICALLY SPEAKING

*Adapted from R. Santa Cruz (2012) for the Understanding Language Project*

This task is to be used with the Formative Assessment Lesson associated with Making Matchsticks during group creation and presentations of posters.

The Language of Mathematics task is provided as a resource to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the questions themselves. Students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

### **Purpose**

This task gives students practice tracking and interpreting target vocabulary words used by their peers during prepared group presentations. Also, it supports students in producing language, since students will be encouraged to prepare and give presentations that incorporate correct use of target words. It is not intended to give students access to the mathematics of the task during the central work on the problem, but rather to provide opportunities for using and understanding key terms when students summarize their work during poster presentations after the central work is complete.

It is crucial that students do this vocabulary work *after* they solve the problem that grounds the meanings for words. Students are likely to use everyday words in their talk while solving a problem in groups and should not be corrected. Instead, the teacher can provide guidance on more formal mathematical terms during a whole-class discussion by providing instruction on the mathematics while modeling the use of more formal vocabulary and ways of talking.

Developing academic language is more than just learning the target or specialized vocabulary of a unit or chapter. Comparative structures

such as “one third of,” “at least,” or “twelve times the amount” are the kinds of syntactic structures that students need to understand and use as they describe their work or make a presentation about their solution for a mathematics task.

### Required for use

- Materials for MARS lesson Making Matchsticks.
- Mathematically Speaking tally charts with target vocabulary words.

### Process details

1. Before using the Mathematically Speaking tally chart, students spend a couple of minutes reviewing their individual solutions to Making Matchsticks (completed in advance of the lesson). Use the following prompts to help students prepare to work with their peers:
  - Summarize the steps that you took to solve the problem.
  - What part of your solution are you most sure about? What part are you least sure about?

Students can share their responses with their peers and reach a consensus as to the “best” solution to the problem. Together, students create posters displaying this joint solution to the problem.

2. After the pairs or groups create the posters, their members present to each other or to the same group (e.g., one member of a pair or group presents to the other members) using the tally sheet to help each other insert the academic language appropriately. The use of recording devices (e.g. iPods, iPads, tape recorders) can help facilitate this process as students can initially record what they want to say and replay their practice presentations. This provides a “safer” way for students to learn how to use academic language and helps ensure student success when they present in front of the whole class.
1. When pairs or groups are finished with their posters, they describe their posters to the whole class. Each student gets one copy of the chart with target vocabulary words entered. As each group describes their poster, each listening student tallies on the chart each time a target word is used. If a target word is not used, the listening students should encourage the presenters to keep



talking, by asking questions or requesting clarifications, until all target words on the list have been used.

2. Students may add words to their chart that come up in their explanations, and then share these with the class at the end of the activity.
3. The lesson continues as described in the MARS lesson outline, with the collaborative analysis of sample student responses.

### **Process outline**

- Each student receives a copy of the handout with the tally chart.
- Each group presents their poster, while other students mark each use of the target vocabulary words on the tally chart, and add words as needed.
- Students are asked to read the list of targeted words silently before the presenters start.
- Stress how important it is for the listeners to pay attention to presenters and listen for targeted words.
- Groups begin their presentations.
- At the end of all presentations, students go over any words they may have added to the given list.

MATHEMATICALLY SPEAKING

Name \_\_\_\_\_

Date \_\_\_\_\_

For each group presentation, mark a tally on the chart every time you hear presenters use one of the target vocabulary words.

Add any words to the chart you hear that are important in the presentations.

Target Word(s)	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Approximate						
Convert						
Cube/Cubic						
Diameter						
Estimate						
Feet						
Formula						
Height						
Inches						
Length						
Measure						
Measurement						
Radius						
Rectangular prism						
Round						
Units						
Volume						
Width						

# Understanding Language

Language, Literacy, and Learning  
in the Content Areas

Understanding Language aims to enrich academic content and language development for English Language Learners (ELLs) by making explicit the language and literacy required to meet the Common Core State Standards (CCSS) and Next Generation Science Standards.

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