Conceptual Category Progressions

Traditional Pathways

2014

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Conceptual Category: Number and Quantity (Modeling Standards denoted with an *)			
Algebra 1	**Geometry**	**Algebra 2**	**Plus Standards**
The Real Number System **Extend the properties of exponents to rational exponents.** N.RN.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5*. N.RN.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents. **Use properties of rational and irrational numbers.** N.RN.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. ***Quantities*** **Reason quantitatively and use units to solve problems.**		***The Complex Number System*** **Perform Arithmetic Operation with Complex Numbers** N.CN.1: Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. N.CN.2: Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **Use complex numbers in polynomial identities and equations** *(+) standards* N.CN.7: Solve quadratic equations with real coefficients that have complex solutions. N.CN.8: Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. N.CN.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	

*N.Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. *N.Q.2: Define appropriate quantities for the purpose of descriptive modeling. *N.Q.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.			

Conceptual Category: Algebra (Modeling Standards denoted with an *)			
Algebra 1	**Geometry**	**Algebra 2**	**Plus Standards**
Seeing Structure in Expressions **Interpret the structure of expressions** ***(linear, exponential, quadratic)*** *A.SSE.1: Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. *b.* Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* **Write expressions in equivalent forms to solve problems** ***(Quadratic and Exponential)*** *A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression		*Seeing Structure in Expressions* ***Interpret the structure of expressions (Polynomial and Rationa)*** *A.SSE.1: Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. *b.* Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* A.SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **Write expressions in equivalent forms to solve problems** *A.SSE.4: Derive the formula for the	

to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* ***Arithmetic with Polynomial and Rational Expressions*** **Perform arithmetic operations on polynomials** ***(Linear and quadratic)*** A.APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.		sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* ***Arithmetic with Polynomial and Rational Expressions*** **Perform arithmetic operations on polynomials** ***(beyond quadratic)*** A.APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	

		A.APR.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.APR.4: Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* A.APR.6: Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	

		(+) standards A.APR.5: Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. A.APR.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	
Creating Equations **Create equations that describe numbers or relationships *(linear, quadratic and exponential for integer inputs)*** *A.CED.1: Create equations and inequalities in one variable and use them to solve problems. *A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes		Creating Equations **Create equations that describe numbers or relationships *(Equations using all available types of expressions, including simple root functions)*** *A.CED.1: Create equations and inequalities in one variable and use them to solve problems. *A.CED.2: Create equations in two or more variables to represent relationships between quantities;	

with labels and scales. *A.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*. *A.CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*. **Reasoning with Equations and Inequalities** **<u>Understand solving equations as a process of reasoning and explain the reasoning</u> *(Master linear, learn as general principle)*** A.REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation		graph equations on coordinate axes with labels and scales. *A.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*. **Reasoning with Equations and Inequalities** **<u>Understand solving equations as a process of reasoning and explain the reasoning</u> *(simple radical and rational)*** A.REI.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **<u>Solve equations and inequalities in one variable</u>**	**Reasoning with Equations and Inequalities** **<u>Solve systems of equations</u>** A.REI.8: Represent a system of linear equations as a single matrix equation in a vector variable. A.REI.9: Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

has a solution. Construct a viable argument to justify a solution method. **<u>Solve equations and inequalities in one variable</u> *(Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions)*** A.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.4: Solve quadratic equations in one variable. a. a.Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to		A.REI.4: Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*. **<u>Represent and solve equations and inequalities graphically</u> *(Combine polynomial, rational, radical, absolute value, and exponential functions)*** A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. *A.REI.11: Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and *y*	

the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **<u>Solve systems of equations</u> *(Linear-linear and linear-quardratic)*** A.REI.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. **<u>Represent and solve equations and inequalities graphically</u> *(Linear and***		$= g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

exponential; learn as general principle) A.REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *A.REI.11: Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.			

Conceptual Category: Functions (Modeling Standards denoted with an *)			
Algebra 1	**Geometry**	**Algebra 2**	**Plus Standards**
Interpreting Functions **Understand the concept of a function and use function notation** ***(learn as a general principle; focus on linear and exponential and arithmetic and geometric sequences)*** F.IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). F.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*		**Interpreting Functions** **Interpret functions that arise in application in terms of the context** ***(Emphasize selection of appropriate models)*** *F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. *F.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **Analyze functions using different**	

<u>Interpret functions that arise in application in terms of the context</u> *(Linear, exponential and quadratic)* *F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. *F.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *F.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **<u>Analyze functions using different representations</u> *(Linear, exponential, quadratic, absolute value, piecewise***		**<u>representations</u> *(Focus on using key features to guide selection of appropriate type of model function)*** *F.IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic*	

defined) *F.IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential		*function and an algebraic expression for another, say which has the larger maximum*. **(+ standards)** *F.IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	

functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay. F.IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*. **Building Functions** **<u>Build a functions that models a relationship between two quantities</u>** ***(linear, exponential and quadratic)*** *F.BF.1: Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying*		**Building Functions** **<u>Build a functions that models a relationship between two quantities</u>** ***(include all functions studied)*** *F.BF.1: Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **<u>Build new functions from existing</u>**	**Building Functions** **<u>Build a functions that models a relationship between two quantities</u>** *F.BF.1: Write a function that describes a relationship between two quantities. c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

exponential, and relate these functions to the model. *F.BF.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **<u>Build new functions from existing functions</u> *(Linear, exponential, quadratic and absolute value)*** F.BF.3: Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.		**<u>functions</u> *(include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types)*** F.BF.3: Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.4: Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.* **(+ standards)** **<u>Build a functions that models a relationship between two quantities</u>** ***(include all functions studied)*** *F.BF.1: Write a function that describes a relationship between two quantities. c. Compose functions. *For example, if T(y) is the temperature in the*	

		atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. **<u>Build new functions from existing functions</u>** BF.4: Find inverse functions. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain. F.BF.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	

Linear, Quadratic and Exponential Models **<u>Construct and compare linear, quadratic, and exponential models and solve problems</u>** *F.LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. *F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *F.LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a		**Linear, Quadratic and Exponential Models** **<u>Construct and compare linear, quadratic, and exponential models and solve problems</u> *(Logarithms as solutions for exponentials)*** *F.LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. *F.LE.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **Trigonometric Functions** **<u>Extend the domain of trigonometric functions using the unit circle</u>** F.TF.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**Trigonometric Functions** **<u>Extend the domain of trigonometric functions using the unit circle</u>** F.TF.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.4: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **<u>Interpret Expressions for functions in terms of the situation they model</u>** ***(Linear and exponential of form*** $f(x)=b^x+k$***)*** *F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.		**<u>Model periodic phenomena with trigonometric functions</u>** *F.TF.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **<u>Prove and apply trigonometric identities</u>** F.TF.8: Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. **(+ standards)** **<u>Extend the domain of trigonometric functions using the unit circle</u>** F.TF.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.4: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	**<u>Model periodic phenomena with trigonometric functions</u>** F.TF.6: Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. *F.TF.7: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **<u>Prove and apply trigonometric identities</u>** F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Conceptual Category: Statistics and Probability			
Algebra 1	**Geometry**	**Algebra 2**	**Plus Standards**
Interpreting Categorical and Quantitative Data **<u>Summarize, represent, and interpret data on a single count or measurement variable</u>** S.ID.1: Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **<u>Summarize, represent, and interpret data on two categorical and quantitative variables</u>** ***(Linear focus, discuss general principle)*** S.ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data	**Conditional Probability and the Rules of Probability** **<u>Understand independence and conditional probability and use them to interpret data</u>** ***(Link to data from simulations and experiments)*** S.CP1: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.2: Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.3: Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	**Interpreting Categorical and Quantitative Data** **<u>Summarize, represent, and interpret data on a single count or measurement variable</u>** S.ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **Making Inferences and Justifying Conclusions** **<u>Understand and evaluate random processes underlying statistical experiments</u>** S.IC.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of*	**Conditional Probability and the Rules of Probability** **<u>Use the rules of probability to compute probabilities of compound events in a uniform probability model</u>** S.CP.8: Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. S.CP.9: Use permutations and combinations to compute probabilities of compound events and solve problems. **Using Probability to Make Decisions** **<u>Calculate expected values and use them to solve problems</u>** S.MD.1: Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.2: Calculate the expected value of a random variable; interpret it as

(including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. **Interpret linear models** S.ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.8: Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.9: Distinguish between	S.CP.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* S.CP.5: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **Use the rules of probability to compute probabilities of compound events in a uniform probability model** S.CP.6: Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	*5 tails in a row cause you to question the model*? **Make inferences and justify conclusions from sample surveys, experiments, and observational studies** S.IC.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.6: Evaluate reports based on data.	the mean of the probability distribution. S.MD.3: Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* S.MD.4: Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **Use probability to evaluate outcomes of decisions** S.MD.5: Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

correlation and causation.	S.CP.7: Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.		a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* S.MD.6: Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7: Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Conceptual Category: Geometry (Modeling Standards denoted with an *)			
Algebra 1	**Geometry**	**Algebra 2**	**Plus Standards**
	Congruence **<u>Experiment with transformations in the plane</u>** G.CO.1: Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.2: Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.3: Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.4: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		

	G.CO.5: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motion** ***(Build on rigid motions as a familiar starting point for development of concept of geometric proof)*** G.CO.6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **Prove geometric theorems** ***(Focus on validity of underlying***		

	reasoning while using variety of ways of writing proofs) G.CO.9: Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*. G.CO.10: Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*. G.CO.11: Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*. **Make geometric constructions** ***(Formalize and explain processes)*** G.CO.12: Make formal geometric		

	constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*. G.CO.13: Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. **Similarity, Right Triangles and Trigonometry** **Understand similarity in terms of similarity transformations** G.SRT.1: Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.2: Given two figures, use the		**Similarity, Right Triangles and Trigonometry** **Apply trigonometry to general triangles** G.SRT.9: Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10: Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

	definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **<u>Prove theorems involving similarity</u>** G.SRT.4: Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* G.SRT.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **<u>Define trigonometric ratios and solve problems involving right triangles</u>** G.SRT.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		

	G.SRT.7: Explain and use the relationship between the sine and cosine of complementary angles. *G.SRT.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Circles** **<u>Understand and apply theorems about circles</u>** G.C.1: Prove that all circles are similar. G.C.2: Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* G.C.3: Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **<u>Find arc lengths and areas of sectors of circles</u>** ***(Radian introduced only as unit of measure)*** G.C.5: Derive using similarity the fact that the length of the arc intercepted		**Circles** **<u>Understand and apply theorems about circles</u>** G.CO.4: Construct a tangent line from a point outside a given circle to the circle.

	by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **Expressing Geometric Properties with Equations** **<u>Translate between the geometric description and the equation for a conic section</u>** G.GPE.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.2: Derive the equation of a parabola given a focus and directrix. **<u>Use coordinates to prove simple geometric theorems algebraically</u>** ***(Include distance formula; relate to Pythagorean Theorem)*** G.GPE.4: Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*		**Expressing Geometric Properties with Equations** **<u>Translate between the geometric description and the equation for a conic section</u>** G.GPE.3: Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

	G.GPE.5: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio. *G.GPE.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. **Geometric Measurement and Dimension** **<u>Explain volume formulas and use them to solve problems</u>** G.GMD.1: Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*. *G.GMD.3: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **<u>Visualize relationships between two-dimensional and three dimensional objects</u>**		**Geometric Measurement and Dimension** **<u>Explain volume formulas and use them to solve problems</u>** G.GMD.2: Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

	G.G.MD.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Modeling with Geometry** **Apply geometric concepts in modeling situations** *G.MG.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). *G.MG.2: Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *G.MG.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).		