**Unit 8: Investigation 1 (4 Days)**

**Introducing Quadratic Functions – Parabolas Everywhere**

***CCSS: A-CED 1, A-CED 2, F-IF4***

**Overview**

Students explore and model data that are approximately quadratic and compare quadratic patterns with linear and exponential patterns. Applications include the distance an object falls over time, the depth of water in a parabolic bowl compared to the radius of the bowl, HIV statistics, and the decline in the Social Security trust fund over time. Students examine simple members of the quadratic family by table and graph and apply their observations to an adaption of Galileo’s falling body problem. Students explore the roles of parameters *a*, *b* and *c* in the standard form of a quadratic function both graphically and numerically, and with the aid of technology, find quadratic equations to model data.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able To Do?**

* Distinguish, given a table of values, between the nonlinear patterns of exponential and quadratic growth
* Make a scatter plot by hand or technology with appropriate scaling and labels and recognize a graph that could be modeled by a quadratic function
* Recognize that for nonlinear growth, the average rates of change will not be constant
* Recognize that for quadratic growth, the average rates of change exhibit linear growth or in other words, the second differences are constant (when Δ*x* is constant)

**Assessment Strategies: How Will They Show What They Know.**

* **Exit Slip 8.1.1** asks students to identify a quadratic function from a table of values and explain why it is quadratic.
* **Journal Entry** asks students explain how to identify linear, exponential and quadratic functions
* **Exit Slip 8.1.2** asks students to match graphs of quadratic functions with values of the parameters*a, b* and *c.*

**Launch Notes**

Depending upon the class, several different activities can take place or the teacher can decide to have all students work on one activity. If the class has already done some motion detector activities and is comfortable with a motion detector, one group could do a bouncing ball activity or drop a beanbag activity. Or the whole class can learn to use a motion detector. **Activity 8.1.3 Rolling Ball & CBR** provides motion detector directions for students.

If time is an issue or if a different kind of experimental activity is desired, a quick but effective activity is to bring in from home a roughly parabolic (interior of the bowl) mixing bowl that you add water to obtain data. See **Activity 8.1.1 Quadratic in the Kitchen**. The key objectives are for students to collect data that is neither linear nor exponential, graph the data, and analyze the behavior.

**Closure Notes**

Hold a class discussion and ensure that students recognize the following key points:

* For linear growth/decay, if Δ*x* = 1, then the next term is always found by adding a constant amount to the prior term.
* For exponential growth/decay, if Δ*x* = 1, then the next term is always found by multiplying the preceding amount by a number (greater than one/between 0 and 1).
* For quadratic growth/decay, if Δ*x* = constant, then Δ(Δ*y*) = constant, thus meaning Δ*y* or first differences are linear.
* The graph of a quadratic function is a parabola.
* The role of the parameters:

*a* determines if the parabola opens up or down as well as stretching or compressing the graph in the vertical direction

*b* moves the graph horizontally as well as contributing to the vertical translation.

*c* has a dual role—the *y*-intercept as well as the vertical translation term

**Teaching Strategies**

1. **Activity 8.1.1** **Quadratics in the Kitchen** involves students collecting data, recording results in a table and making a graph. HIV data from the Centers for Disease Control are found in **Activity 8.1.2 Modeling HIV Data**.

Here is a sample set of data for **Activity 8.1.1** **Quadratics in the Kitchen**.

|  |  |
| --- | --- |
| **Radius (cm)** | **Depth of water in bowl (cm)** |
| 5.9 | 1.1 |
| 6.2 | 1.8 |
| 6.95 | 2.9 |
| 7.55 | 4.2 |
| 8.2 | 6 |

**Activity 8.1.3 Rolling Ball & CBR 2** requires students to use a motion detector to collect data on the distance an object travels as it is dropped. Stations might be set up around the room for students to these three activities.

1. After students have collected, organized and graphed their data, define a quadratic function as one whose defining expression can be put in the form of $y=ax^{2}+bx+c$ where *a, b* and *c* are real numbers and *a* cannot be 0.

Introduce the triangular numbers 1, 3, 6, 10 either using dots in triangular arrays or use blocks to generate the table below.

Find the changes in the number of blocks to prove that the table is not defining a linear function. Find ratios to demonstrate there is no common ratio, so the function is not exponential. Then define second differences (differences of consecutive Δ*y*) and find the second differences with the class to show that they are constant.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Number of rows (*x*)** | **Dots or blocks** **(*y*)** | **First Difference ∆*y*** | **Ratios** | **Second Difference ∆(∆*y*)** |
| 1 | 1 | ---- | --- | ---- |
| 2 | 3 | 2 | 3 | ---- |
| 3 | 6 | 3 | 2 | 1 |
| 4 | 10 | 4 | 5/3 | 1 |
| 5 | 15 | 5 | 3/2 | 1 |

Then use **Activity 8.1.4 Quadratic Functions by Table** to study additional tables of some members of the quadratic family and to do some additional graphing using the table of values in the activity. Students may need assistance finding second differences.

Follow up with **Activity 8.1.5 Social Security Trust Fund**. In this activity students will see that the first differences are decreasing so the function is not linear. The second differences, however, are not constant, so a quadratic function is not a perfect fit. Nevertheless, a quadratic function is a good fit, since the shape of the graph is roughly parabolic.

Students may not have enough time to finish these activities in class. In particular, the graphs for **Activity 8.1.4** may be assigned for homework. In anticipation of the next lesson you may ask students to research the heights of the tower of Pisa and the tallest buildings in the world today. The Leaning Tower is 183- 186 feet—it leans so it depends upon what side you are on. The tallest building is in Dubai (as of May 2008) and is 2717 feet tall. The CN Tower in Toronto is 1815 feet. Prior to the Khalefa Building going up in Dubai, the CN Tower was the tallest. Other tall buildings are currently under construction.

**Differentiated Instruction (For Learners Needing More Help)**

If students need practice plotting points they should be asked to make at least some graphs by hand. Otherwise encourage students to use calculators to make scatterplots by entering data in lists L1 and L2 and using the STAT PLOT feature.

**Differentiated Instruction (For Enrichment)**

Ask students to find quadratic functions that fit the data in **Activities 8.1.3 and 8.1.5**. This will require that they experiment with values for *a*, *b*, and *c*. They can test their functions by entering them into the Y= menu and observing how close they come to the points of the scatter plot. Once they have a good fit they can check their result against the equation the calculator gives when asked to produce a quadratic regression (STAT CALC 5 on the TI-84).

At this point you may use **Exit Slip 8.1.1,** which asks students to identify quadratic functions from tables.

**Journal Entry**

How can you tell from a table of values whether a function is linear, exponential, or quadratic?

1. **Activity 8.1.6 Exploring the Parameters of** $y=ax^{2}+bx+c$is a systematic exploration of how the parameters *a, b* and *c* affect the behavior of the graph. This activity is 9 pages long, but it can be done in one class period if you have students work in groups and break up the tasks as suggested in the box below.

**Group Activity**

Have students work in pairs to work on **Activity 8.1.6 Section 1** (Identifying *a, b,* and *c*) and then share answers with the class. Work through case 1 of Section 2 with the entire class to model the process of observing the effect of a change in parameter. Then divide the class into four groups, assigning each group one of pages 3 through 6. Have each group report their findings to the whole class. This will complete the investigation of the roles of *a* and *c.* Then address section 4 (pages 7 and 8), which deals with the more problematical role of *b,* through a whole class discussion or again assign students to work in groups. Depending upon the amount of class time remaining, Sections 5 and 6 (page 9) may be assigned for homework.

In a class discussion emphasize that the graph of every quadratic function is a parabola. If a function has a parabola-like shape it might not be a quadratic function. Graphs of other even-degree polynomial functions, such as $f\left(x\right)=x^{6}$, look like parabolas but are not parabolas.

You may also draw out these features of quadratic functions:

* A parabola is a U-shaped curve that **opens up** or **opens down**. The value of *a* tells us the direction.
* There is a lowest point or highest point called the **vertex**. We need the *y-*coordinate of the vertex to determine the **range** of a quadratic function.
* The **domain** of a quadratic function is all real numbers.
* When *a* > 1 or *a* < -1 the graph of $y=ax^{2}$ will be **more narrow** than the graph of $y=x^{2}$. If –1 < *a* < 1, the graph of $y=ax^{2}$ will be **wider** than the graph of $y=x^{2}$.
* For functions of the form $y=ax^{2}$, the origin, (0, 0), is the vertex, *y*-intercept and the *x*-intercept.
* For functions of the form $y=ax^{2}+c$, (0, *c*) is the *y*-intercept and the vertex. When *a* > 0, if *c* > 0 there are no *x*-intercepts, and if *c* < 0 there are two *x*-intercepts.
* For functions of the form $y=ax^{2}+bx$, the *y*-intercept and one of the *x*-intercepts is (0,0). Furthermore, if *b* > 0 the other *x*-intercept lies to the left of the *y*-axis, while if

*b* < 0, the other *x*-intercept lies to the right of the *y*-axis. Later in the unit we will address finding *x*-intercepts.

**Differentiated Instruction (For Learners Needing More Help)**

The above bullets contain a lot of information, which will not be immediately absorbed by most students. A bulletin board with some of this information presented in graphical form will be helpful.

By now students should appreciate that there is a vertical **axis of symmetry** and that if we know the coordinates of the vertex, defining the equation of the axis of symmetry is simple. The vertex form the function is addressed in the next investigation.

Students often feel that because the graph of $y=x^{2}+2$ is nested within the graph of $y=x^{2}$, that it is more narrow. Talk about this to dispel this misconception. Since *a* = 1 these parabolas have exactly the same shape and one could be superimposed upon the other. You may demonstrate this by cutting parabolas out of felt paper and sliding one to fit on the other.

1. After summarizing the results of the investigation of parameters in Activity 8.1.6 have students work on **Activity 8.1.7 Galileo in Dubai** in which they apply what they have learned to derive an equation for a falling object. This is also an opportunity to review the concepts of domain and range and emphasize that these are often restricted in a real life application. Question 13, which asks when the object will hit the ground, can be used to motivate the need to solve quadratic equations, which is taken up later in the Unit.

Bring closure to the activities of this investigation and set up a need for another form for a quadratic equation – one that highlights the vertex.

At this point you may use **Exit Slip 8.1.2**, which asks students to match graphs with values of the parameters *a*, *b*, and *c*.

**Resources and Materials**

* **Activity 8.1.1** Quadratics in the Kitchen
* **Activity 8.1.2** Modeling HIV Data
* **Activity 8.1.3** Rolling Ball & CBR-2
* **Activity 8.1.4** Quadratic Functions by Table
* **Activity 8.1.5** Social Security Trust Fund
* **Activity 8.1.6** Exploring the Parameters of y=ax^2+bx+c
* **Activity 8.1.7** Galileo in Dubai
* **Exit Slip 8.1.1** Is it Quadratic?
* **Exit Slip 8.1.2** Matching Parabolas with Parameters
* Bowls, string, metric rulers, and water will be needed for **Activity 8.1.1**
* Calculators, CBR-2, and beanbags will be needed for **Activity 8.1.3**
* Computer access for the launch, class set of graphing calculators, activity handouts, [www.education.TI.com](http://www.education.TI.com)
* Quadratic Function Applications: [www.math.LSA.umich.edu/courses/105/m105\_f05\_h4.pdf](http://www.math.LSA.umich.edu/courses/105/m105_f05_h4.pdf)