**Unit 7: Investigation 5 (3 Days)**

**EXPONENTIAL PATTERNS AS PERCENT CHANGE**

CCSS: A-SSE 1b. A-SSE 3c, F-IF 8b, F-LE 1c, F-LE 5

**Overview**

Exponential growth or decay is often described in terms of percent increase or percent decrease. In this investigation students explore the relationship between percent rate of change (percent change) and the growth or decay factor they have learned to associate with the parameter *b* in the general form of the exponential function $f\left(x\right)=ab^{x}$. The investigation concludes with a discussion of compound interest and the concepts of doubling time and half-life.

**Assessment Activities**

 **Evidence of Success: What will students be able to do?**

* Given a percent rate of change students will be able to determine the growth or decay factor and write an explicit equation for an exponential function.
* Given an exponential function students will be able to determine the percent rate of change and the growth or decay factor.
* Students will apply their understanding of exponential functions to the computation of compound interest.

 **Assessment Strategies: How will they show what they know?**

* **Exit Slip 7.5.1** assesses students’ understanding of the relationship between percent rate of change and growth and decay factors.
* **Journal Prompt** asks students to explain how to find percent rate of change given a growth or decay factor.
* **Exit Slip 7.5.2** asks students to calculate the value of an investment with compound interest.

**Launch Notes**

Begin by showing a five-minute video clip from the Futures Channel about the US Fish & Wildlife Service efforts to reestablish a population of black-footed ferrets: <http://thefutureschannel.com/videogallery/the-blackfooted-ferret/>

This leads to a discussion about percent change in **Activity 7.5.1.**

**Closure Notes**

Ask students to suggest ways they can remember the distinction between a percent rate of change such as 6% and a growth factor such as 1.06.

**Teaching Strategies**

1. After introducing the video, ask the students what it means for a population to grow at a rate of 10% per year. Ask them to think about whether that type of growth will be linear, exponential, or something else. Then introduce **Activity 7.5.1 Percents and Percent Change.**

The first two pages of this activity constitute a review of calculations with percents. Depending upon the background of your students you may want to spend more or less time on these pages.

**Differentiated Instruction (For Learners Who Need More Help)**

This is a good opportunity for students who need a refresher to review the relationship between fractions, decimals, and percents.

The key ideas of this activity are developed on pages 3 and 4. That is, a growth rate of *r*% per time period is equivalent to a growth factor of (100 +*r*)%. Similarly if a quantity is decreasing at a rate of *r*% per time period, this is equivalent to a decay factor of (100 – *r*)% per year. Multiplying by the growth or decay factor is more efficient that calculating the amount of increase or decrease and then adding to or subtracting from the starting amount. Furthermore finding the growth or decay factor easily leads to an explicit equation for the associated exponential function as shown in subsequent activities.

Extension of the concept of percent change to exponential growth or decay and exponential functions is developed in **Activity 7.5.2 Percent Change and Exponential Functions.** Two contexts are presented here: the growth of cell phone use in the late 20th century and the decimation of a population of insects due to an insecticide. Students begin with tables and graphs and then construct equations for the exponential functions. The last page explores the relationship between exponents and repeated multiplication and the algebraic form of the exponential function.

After completing the first two activities, **Exit Slip 7.5.1** may be used to assess students understanding of the relationship between percent change and exponential functions.

1. In **Activity 7.5.3 Percent Change Situations** students are given situations in which a percent rate of change (percent change) is given. They determine whether the situation is best modeled with a linear or an exponential function and whether the situation represents exponential growth or decay. They then write an expression for the exponential function and use the function to find a particular value of the dependent variable.

**Activity 7.5.4 Modeling Exponential Functions: What Is the Percent Change?** reverses the process developed earlier. Here students are given a table of values or an explicit formula from which they can determine the growth or decay factor. This then enables them to find the percent rate of change (percent change).

**Journal Prompt**

If you are given an exponential function of the form $f\left(x\right)=ab^{x}$, how can you determine the percent rate of change? Give two examples, one involving exponential growth and the other exponential decay.

1. An important application of percent rate of change is compound interest. This is developed in **Activity 7.5.5 Compound Interest.** The activity begins by having students fill out a table that shows step by step how an investment with compound interest grows. They should notice that the amount of interest added each year grows since the base on which the percent is calculated is also growing. By now they should readily realize that a growth rate of 3% per year is equivalent to a growth factor of 1.03. They then use the constant multiplication feature of the graphing calculator to generate the same sequence of values in column 4. You may also want to show how the same table may be generated on a spreadsheet.

Compound interest, which is an example of exponential growth, is then contrasted with simple interest, which is an example of linear growth. The activity concludes by introducing interest that is compounded more frequently than once a year and the concept of annual yearly rate (APY). These are important concepts for students to become familiar with, as banks will often refer to APY in describing investments.

**Differentiated Instruction (Enrichment)**

Encourage students to explore the effect of increasing the frequency of compounding. For example, what is the APY for an investment at a 6% rate of interest if the compounding (a) once a year, (b) twice a year, (c) four times a year, (d) monthly, (e) daily? Then challenge students to find a formula for the value of an investment compounded *n* times a year for *t* years with an initial principle *p* and an annual rate of interest *r*.

**Exit Slip 7.5.2** asks students to calculate the value of an investment with compound interest.

As a final activity in this investigation, students explore the concepts of doubling time and half-life in **Activity 7.5.6.** They may also discover the “Rule of 72.” This activity is particularly suited for group work.

**Group Activity**

Have students work in small groups to complete **Activity 7.5.6 Doubling Time and Half-Life.** They can discover the Rule of 72 by dividing up the work, each taking a different rate of interest to figure out the doubling time.

The Rule of 72 was used for centuries by investors to make a quick calculation of doubling time. It only gives approximate results and is most accurate for interest rates in the range 4-18%. See [http://financial-dictionary.thefreedictionary.com/rule+of+72](http://financial-dictionary.thefreedictionary.com/rule%2Bof%2B72). A more detailed mathematical explanation for the teacher may be found at <http://en.wikipedia.org/wiki/Rule_of_72>

**Resources and Materials**

* **Activity 7.5.1 Percents and Percent Change**
* **Activity 7.5.2 Percent Change and Exponential Functions**
* **Activity 7.5.3 Percent Change Situations**
* **Activity 7.5.4 Modeling Exponential Functions: What is the Percent Change?**
* **Activity 7.5.5 Compound Interest**
* **Activity 7.5.6 Doubling Time and Half-Life**
* **Exit Slip 7.5.1 Percent Change**
* **Exit Slip 7.5.2 Compound Interest**
* Video cliphttp://thefutureschannel.com/videogallery/the-blackfooted-ferret/
* Graphing Calculators
* Bulletin board for key concepts