**Unit 5: Investigation 6 (3 Days)**

**Piecewise Functions**

***CCSS:*** 8-SP 1; 8-SP 2; 8-SP 3; S-ID 6 a, c; S-ID 7, F-IF 7b

**Overview**

Students will explore situations in which the data are best represented by a piecewise linear function, will fit a line to each section of the data set, and will use the lines to make predictions.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

Students will be able to identify two points on each line segment and use them to calculate the equation of the line that contains that segment. They will be able to identify the domain for which the line segment fits the data, and write the piecewise function given the graph. Students will be able to create a story that describes a piecewise graph.

**Assessment Strategies: How Will They Show What They Know?**

**Exit Slip 5.6** asks students to create a piecewise function to represent the height of an elevator during an elevator ride.

**Journal Entry 1** probes students’ reaction to encountering piecewise functions and their comfort level with this new concept.

**Journal Entry 2** asks students to explain how to write a rule for a piecewise function if they are given a graph.

**Launch Notes**

Display **Activity 5.6.1 Swimming Records**. This activity will provide a smooth transition from scatterplots to piecewise functions. Focus the students’ conversations on how the data is similar to and different from the data sets previously studied in Unit 5.

**Closure Notes**

Ask students to describe real world situations for which piecewise functions are good models. Draw out the fact that in each situation they have encountered the slope of the line (that is the rate of change) is different for the separate pieces of the graph. You may want to go back to specific examples such as the triathlon (speeds differ for different events) or the dog food activity (rates differ depending upon the size of the dog).

**Teaching Strategies**

1. Begin with **Activity 5.6.1 Swimming Records.** Make sure that students understand that the data refer to individuals who have set a new swimming record, not the individual with the best time during that particular year. The gap between 1936 and 1952 results from the fact during that time period no one broke the record set by Willy den Ouden in 1936. Have students fit a trend line by hand to the graph on page 1. Statistically it gives a fairly good fit; the regression line shows a strong negative correlation with *r* = –0.96. You may want to show the regression line with your calculator on the overhead projector. Point out that points are not evenly distributed around the regression line: those at either end of the data set lie above the line and those in middle lie below. This suggests that two lines might produce a better fit than just one line. Ask students to use the graph on page 2 to fit two trend lines by hand. Go over the questions on this page in class to introduce students to the concept of a piecewise function.

**Technology Tip:** If you decide to use the calculator to show regression lines for each piece of the data, you may enter the data into lists L1 and L2. Then copy L1 into L3 and L5 and copy L2 into L4 and L6. In L3 and L4, delete ordered pairs that have *x*-values (years since 1910) greater than 26. In L5 and L6, delete ordered pairs that have *x*-values (years since 1910) less than or equal to 26. Then find the regression lines for L4 vs. L3 and for L6 vs. L5.

Use **Activity 5.6.2 Paychecks & Triathlons** to strengthen student understanding of piecewise functions. Here the data are perfectly linear, so each piece of the function will be a perfect fit. When discussing the domain, you can either have the students write it in terms of *x* (i.e. 0 ≤  *x* ≤ 40 or *x* > 40) or you may have students use words (i.e. working between 0 and 40 hours or working more than 40 hours). You may do the first part of this activity (Paychecks) in class and assign the second part (Triathlons) for homework.

**Differentiated Instruction (For Learners Needing More Help)**

Have students write down the steps for writing an equation and how to determine the domain for each part of a piecewise function on a notecard. Allow them to use this card throughout the investigation.

**Differentiated Instruction (Enrichment)**

Students may research the history of the triathlon and the different kinds of races and their distances. Ask them to consider how realistic it is to model each segment of the race with a linear function, when in fact the racer’s speed may very. Students who are interested may be encouraged to participate in a local triathlon event <http://www.trifind.com/ct.html>. If they do, they may construct a piecewise function to model their performance.

1. **Activity 5.6.3 Dog Food** presents another real world situation that can be represented by a piecewise function. This activity can be assigned for students to complete in groups. You could assign one type of dog (small, medium, or large) to each group. (If you have more than three groups, two or more groups will have the same type of dog.)

After the groups have derived an equation for their dog type, they can put their answers to questions (2), (3), and (5) on the board. In a whole class discussion, have students figure out how to represent the function in the format shown in question (8). Then have students return to their groups to answer the remaining questions.

**Group activity**

**Activity 5.6.3** is particularly suited for grouping high level students with low level students so that someone in the group will remember how to find an equation of a line given two points. As you circulate, make sure that all students in each group understand the derivation of their equation.

An optional follow up to **Activity 5.6.3** is **Activity 5.6.4 Feeding the Birds**. This may be started in class or part of it may be assigned as homework. Several features make this activity a bit more challenging. First, the dependent variable is not a traditional unit of measure; in this case our unit is the amount of food in a birdfeeder. Second, students need to reason about how the height of the bird feed determines the number of perches available, which in turn determine the rate at which the height decreases. Third, when calculating the slope, fractions will appear in the numerator. Since the units for *y* are fractions, the equations should be in fractions, not rounded decimals. The first five questions on the 4-perch feeder are more scaffolded than the remaining ones on the 6-perch feeder. The extension questions may be assigned for enrichment.

**Differentiated Instruction (Enrichment)**

Students can build a physical model with rice as suggested in the Extension questions. They should compare how well the mathematical model corresponds with the physical one.

If you find that your students are doing very well with piecewise functions, then give **Exit Slip 5.6** after the first three activities. If your class has a good grasp of piecewise functions, then you may be able to skip the third day. At this point you may use the following Journal Entry.

**Journal Entry 1**

Reflect on your participation in class today and complete two or more of the following statements:

 I learned that I…

 I was surprised that I…

I noticed that I ….

I discovered that I…

I was pleased that I…

1. Two additional activities are appropriate for a third day. Students create and interpret graphs of piecewise functions, write rules for piecewise function, evaluate the functions, and finally write stories that correspond to a graph.

**Activity 5.6.5 Bike Tours** presents two scenarios involving piecewise functions and distance-time graphs. You may choose to use one or both scenarios. In class the activity may be done as a teacher-led lesson or as a group work activity. If Scenario 1 is done in class, Scenario 2 may be assigned for homework. In question (23), stress to the students that “45 minutes” needs to be converted to hours because the piecewise function rule is in terms of hours. Set up a ratio 45/60 and have them simplify to either ¾ or 0.75 hours.

**Differentiated Instruction (For Learners Needing More Help)**

Remind students of the graphs that they created in Unit 4 using a motion detector. Many of them could have been modeled by piecewise functions. You might set up the motion detector and ask someone to “walk a graph” that would look like Jackie’s bike trip.

**Activity 5.6.6 Creating Stories** provides additional practice with distance-time graphs and has students create their own story and write a function to match it.

**Journal Entry 2**

If you are given a graph of a piecewise function, explain how you would find the function rule and the domain for each piece of the function.

**Resources and Materials**

* **Activity 5.6.1** Swimming Records
* **Activity 5.6.2** Paychecks & Triathlons
* **Activity 5.6.3** Dog Food
* **Activity 5.6.4** Feeding the Birds
* **Activity 5.6.5** Bike Tours
* **Activity 5.6.6** Creating Stories
* **Exit Slip 5.6 -** Elevator Ride
* Bulletin board for key concepts
* Individual white boards, markers and erasers
* Clear cylindrical container and rice