

Mathematics Instructional Cycle Guide

CCSS.MATH.CONTENT.HSA.REI.C.5

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Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Insert the cluster heading and Content Standard(s) here.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP 1 - Problem Solving

MP 4 - Model with Mathematics

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (**Page 2**).
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (**Pages 3 – 6**).
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (**Pages 7 - 12**)
- Supporting lesson materials (**Pages 17 - 18**)
- Precursory research and review of standard **CCSS.MATH.CONTENT.HSA.REI.C.5** and assessment items that illustrate the standard (**Pages 13 - 16**)

HOW TO USE THIS DOCUMENT

- 1) Before the lesson, administer the (**Shopping at the Mall**) [*Mathematical Checkpoint*](#) individually to students to elicit evidence of student understanding.
- 2) Analyze and interpret the student work using the [*Student Response Guide*](#)
- 3) Use the next steps or **follow-up lesson plan** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
- 4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **27"x 32"** pad of paper. Each piece of paper is divided in to 4 sections
- **Projector**
- **Crayons/Colored Pencils**
- **Rulers**
- **Graph paper**
- **Lined paper**
- **8 ½" x 11"** paper cut in half width-wise, to be used for Exit Slip
- **TI-84** if available, but not necessary

TIME NEEDED

Shopping at the Mall administration: **10 minutes**

Follow-Up Lesson Plan: **2 periods**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

**Step 1: Elicit evidence of student understanding
Mathematical Checkpoint**

Question(s)	Purpose	
<p>Anna and Jasmine went to the mall together. One store had all DVDs marked down to a special price. At the shoe store, they both bought the same style sneaker. In fact, Jasmine even bought an additional pair of the same sneaker for her sister. Anna spent \$95 on 3 DVDs and 1 pair of sneakers. Jasmine spent \$142 on 2 DVDs and 2 pairs of sneakers.</p> <p>Write and solve a system of equations to find the cost of a DVD and the cost of a pair of sneakers.</p> <p>Interpret the solution in context.</p>	<p>CT Core Standard:</p>	<p>CCSS.MATH.CONTENT.HSA.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.</p>
	<p>Target question addressed by this checkpoint:</p>	<p>Can students write equations using two variables to model a real-world situation? Can students solve the system using a method of their choice (graphing, elimination or substitution)? Can students interpret the solution in the context of the problem?</p>

Step 2: Analyze and Interpret Student Work
Student Response Guide

Got It

Anna and Jasmine went to the mall together. One store had all DVDs marked down to a special price. At the shoe store, they both bought the same style sneaker. In fact, Jasmine even bought an additional pair of the same sneaker for her sister. Anna spent \$95 on 3 DVDs and 1 pair of sneakers. Jasmine spent \$142 on 2 DVDs and 2 pairs of sneakers.

$x = \text{cost of sneakers}$ $y = \text{cost of DVD}$

Write and solve a system of equations to find the cost of a DVD and the cost of a pair of sneakers.

$95 = 3x + 1y$ $142 = 2x + 2y$

How much did each DVD cost? How much was a pair of sneakers?

Each DVD cost \$12 $x = 12$
Each sneaker pair \$59 $y = 59$

$-9 \cdot 95 = 3x + 11$
 $2(142) = 2x + 2y$

$-190 = 6x - 2y$
 $286 = 6x + 6y$
 $\frac{236}{4} = \frac{4y}{4}$
 $59y$
 $12x$

$95 = 3(12) + 1(59)$ $142 = 2(12) + 2(59)$
 $95 = 36 + 59$ $142 = 24 + 118$
 $95 = 95$ $142 = 142$ ✓

Developing

Anna and Jasmine went to the mall together. One store had all DVDs marked down to a special price. At the shoe store, they both bought the same style sneaker. In fact, Jasmine even bought an additional pair of the same sneaker for her sister. Anna spent \$95 on 3 DVDs and 1 pair of sneakers. Jasmine spent \$142 on 2 DVDs and 2 pairs of sneakers.

Write and solve a system of equations to find the cost of a DVD and the cost of a pair of sneakers.

$95 = 3x + y$ $142 = 2x + 2y$

How much did each DVD cost? How much was a pair of sneakers?

$95 = 3x + y$ $95 = 3(9.50) + y$
 $142 = 2x + 2y$ $95 = 28.5 + y$

sneakers = 66.50
DVD = 9.50

$\frac{x}{0} \begin{array}{r} 3 \cdot 180 = 6x - 2y \\ 142 = 2x + 2y \\ \hline -38 = -4x \\ -4 \quad -4 \\ \hline 9.50 = x \end{array}$

$\$9.50 = x$

Getting Started

Anna and Jasmine went to the mall together. One store had all DVDs marked down to a special price. At the shoe store, they both bought the same style sneaker. In fact, Jasmine even bought an additional pair of the same sneaker for her sister. Anna spent \$95 on 3 DVDs and 1 pair of sneakers. Jasmine spent \$142 on 2 DVDs and 2 pairs of sneakers.

$x = \text{DVD}$
 $y = \text{sneakers}$

Write and solve a system of equations to find the cost of a DVD and the cost of a pair of sneakers.

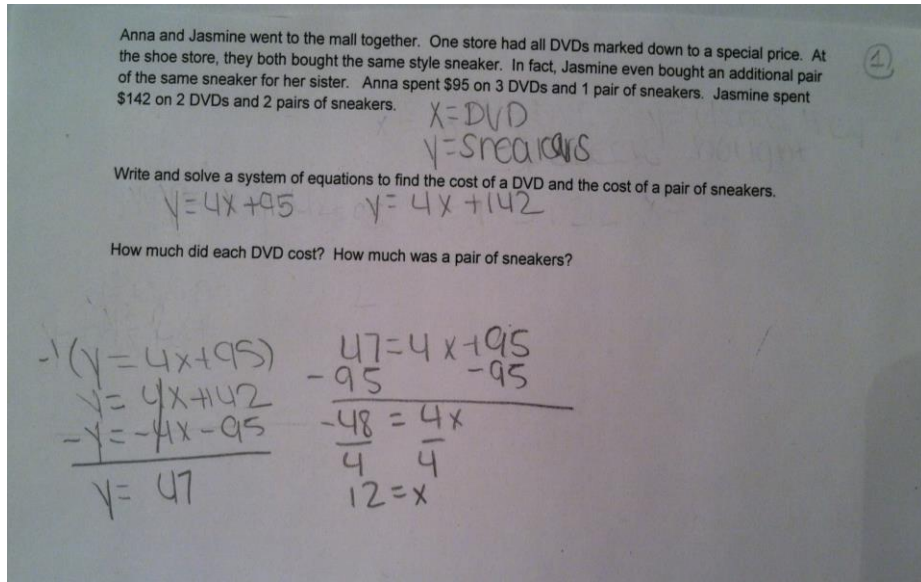
$y = 4x + 95$ $y = 4x + 142$

How much did each DVD cost? How much was a pair of sneakers?

$-1(y = 4x + 95)$ $47 = 4x + 95$
 $y = 4x + 142$ -95
 $-y = -4x - 95$ $-48 = 4x$
 $y = 47$ $4 \quad 4$
 $12 = x$

Getting Started

Student Response Example



Indicators

- The student is unable to identify the variables and/or the constants creating inaccurate equations.
- The student is unable to identify the relationship between the variables and the constants.
- The student is unable to identify and describe the solution, including whether the solution is reasonable.
- The student does not understand the concept of elimination when working with opposite values.
- The student neglects to change signs of all terms when distributing and/or does not understand dividing opposite signs.

In the Moment Questions/Prompts

- Q: Does your answer seem reasonable?
- Q: What value do you get when subtracting y from y? Or 4x from 4x?
- Q: What can you do to see if your solution is correct?

Closing the Loop (Interventions/Extensions)

Since a solution may be difficult to see when graphing, this video offers examples on how to solve systems of equations using the substitution method. <http://learnzillion.com/lessons/1362-solve-systems-of-linear-equations-using-substitution>

View how to convert a real-world situation in to an equation. This video offers steps on how to write an equation with variables. <http://ctdreamteam.learnzillion.com/lessons/1511-convert-a-realworld-situation-into-an-equation>

Developing

Student Response Example

Anna and Jasmine went to the mall together. One store had all DVDs marked down to a special price. At the shoe store, they both bought the same style sneaker. In fact, Jasmine even bought an additional pair of the same sneaker for her sister. Anna spent \$95 on 3 DVDs and 1 pair of sneakers. Jasmine spent \$142 on 2 DVDs and 2 pairs of sneakers.

Write and solve a system of equations to find the cost of a DVD and the cost of a pair of sneakers.

How much did each DVD cost? How much was a pair of sneakers?

$$95 = 3x + y \quad 142 = 2x + 2y$$

$$95 = 3x + y - 2 \quad 95 = 3(9.50) + y$$

$$142 = 2x + 2y \quad 95 = 28.5 + y$$

$$-3180 = -6x - 2y \quad 28.5 - 28.5$$

$$142 = 2x + 2y$$

$$-38 = -4x$$

$$-4 \quad -4$$

$$\$9.50 = x$$

sneakers = 66.50
DVD = 9.50

Indicators

- Variables and constants are correctly identified.
- Equations are created correctly.
- The solution has not been checked.
- Computational mistakes occurred.
- The student only uses one equation to find the answer. The solution must be true for both equations.
- The solution is stated clearly.

In the Moment Questions/Prompts

- Q: What is the solution for this system of equations?
- Q: Is the solution true for both linear equations?
- Q: What can you do to see if your solution is correct?
- Q: Did you check your solution?
- Q: What is a non-example of a solution and what would that mean?

Closing the Loop (Interventions/Extensions)

Using the elimination method, systems of equations are solved using 2 multipliers.

<http://learnzillion.com/lessons/162-solve-systems-of-equations-using-elimination-4>

In this video, it is explained what it means for an ordered pair to be a solution to a system of linear equations.

<http://ctdreamteam.learnzillion.com/lessons/4039-decide-whether-a-point-is-a-solution-to-a-pair-of-linear-equations>

Got it

Student Response Example

Anna and Jasmine went to the mall together. One store had all DVDs marked down to a special price. At the shoe store, they both bought the same style sneaker. In fact, Jasmine even bought an additional pair of the same sneaker for her sister. Anna spent \$95 on 3 DVDs and 1 pair of sneakers. Jasmine spent \$142 on 2 DVDs and 2 pairs of sneakers.

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Write and solve a system of equations to find the cost of a DVD and the cost of a pair of sneakers.

$$95 = 3x + 1y \quad 142 = 2x + 2y$$

How much did each DVD cost? How much was a pair of sneakers?

Each DVD cost \$12 $x = 12$
Each sneaker pair \$59 $y = 59$

$$\begin{array}{r} -9(95 = 3x + 1) \\ 3(142 = 2x + 2) \\ \hline -190 = 6x - 2y \\ 426 = 6x + 6y \\ \hline 236 = 4y \\ 4 \quad 4 \\ \hline 59y \\ 12x \end{array}$$

$$\begin{array}{ll} 95 = 3(12) + 1(59) & 142 = 2(12) + 2(59) \\ 95 = 36 + 59 & 142 = 24 + 118 \\ 95 = 95 \quad \checkmark & 142 = 142 \quad \checkmark \end{array}$$

Indicators

- Variables and constants have been correctly identified.
- No computational errors exist.
- Equations represent the situation correctly.
- The solution has been checked and is correct.
- The solution is identified and correct.
- The solution is reasonable.

In the Moment Questions/Prompts

- Q: What is the solution to this system of equations?
- Q: What is a non-example of a solution and what would that mean?
- Q: Would it make sense to have a negative answer in this situation? Why or why not?
- Q: Graph this situation. What does the slope and y-intercept represent?
- Q: How would you solve this system of equations using substitution?
- Q: Are there any constraints on the solution?

Closing the Loop (Interventions/Extensions)

Learn how to represent constraints on solutions by writing linear equations. This video explains why a negative value is not a solution to this real-life situation.
<http://ctdreamteam.learnzillion.com/lessons/2953-represent-constraints-on-solutions-using-linear-equations#quickcode-modal>

Learn how to represent constraints on solutions by writing linear inequalities.
<http://ctdreamteam.learnzillion.com/lessons/2908-represent-constraints-on-solutions-using-linear-inequalities>

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction

Lesson Objective:	Students will be able to solve a System of Equations by graphing, substitution or elimination.
Content Standard(s):	<u>CCSS.MATH.CONTENT.HSA.REI.C.5</u> Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Targeted Practice Standard :	<i>MP 1 – Problem solving</i> <i>MP 4 – Model with mathematics</i>

Mathematical Goals	Success Criteria
<ul style="list-style-type: none"> Understand that a solution for a system of equations is a solution for all given equations. Understand an expression for y in terms of x is obtained from one of the equations, and should form a true statement when substituted into another equation of the system. Understand a system of equations can be solved using various methods. Identify the solution of a system of equations. Understand how to check your solution properly solving a system using any of the 3 methods. 	<p><i>Create and solve a system of equations with 2 linear equations representing the situation, and identify variables.</i></p> <p><i>Interpret the solution, as it pertains to the problem, as an ordered pair, and when viewing a graph or table.</i></p>

Launch (Probe and Build Background Knowledge)

Purpose: *Create the same value in 3 different ways then set all three equal to each other using substitution.*

1. *Ask a student to pick a value from 1 – 10. Write the value on the board.*
2. *Ask another student to represent the value as a product of two numbers. Write the response slightly above to the left of the number from the previous step.*
3. *Ask another student to represent the value from step 1 as a sum of 2 numbers. Write the response slightly above to the right of the number from step 1.*
4. *What does the product equal? Draw an equal sign between the product and the value.*
5. *What does the sum equal? Draw an equal sign between the sum and the value.*
6. *What can we conclude about the value of the product and the sum?*
7. *Write the 3 equations that were created: the product = value, the sum = value, the product = the sum*

(See example below)

Example:

$$\begin{array}{ccc} \mathbf{2(5)} & = & \mathbf{7 + 3} \\ & = & \mathbf{10} \end{array}$$

Instructional Task

Purpose: *Create and solve a system of equations using graphing, substitution, or elimination.*

Engage (Setting Up the Task)

Students are to be in groups of 3 or 4 based upon the results of the Checkpoint. One student from each skill level. Be sure to assign a student at the low level with another student of the high level. The lowest level student is the “Solver”; the middle level student is the “Recorder” and the highest level student is the “Checker” asking questions to confirm accuracy.

Each group receives one piece of paper which has been folded in quarters. Students are to write all the names of those in the group at the top of the page. Beginning with the top left square, write “Word Problem”. Moving clockwise the next square is titled “System of Equations”. Moving to the right, the next square is titled, “Table” and the last square is titled, “Solution and Check”. (Template included at end in Teacher Material.)

As a model, provide the following example:

The following word problem will be projected on the board:

Thirty tickets were sold to the school Art Show on Friday. Student tickets were \$2, non-student tickets were \$5. The school made \$90. How many student tickets were sold? How many non-student tickets were sold?

This word problem is to be written on the top left square of the teacher’s paper.

Ask the whole class:

What are the variables we are identifying?

How can we create equations for our System?

What are the equations for our System?

Write the equations and identify the variables in the top right square:

x is the number of student tickets sold

y is the number of non-student tickets sold

$$x + y = 30$$

$$2x + 5y = 90$$

In the square below the newly created System, begin to build a table:

$x + y = 30$		$2x + 5y = 90$	
x	y	x	y
0	30	0	18
1	29	1	17.6

Ask the students why would we begin with $x = 0$ and not a negative number?

Why isn't a y -value of 17.6 reasonable?

In the table, what will we see that will tell us the lines are intersecting?

Continue to build the table until $x = 20$. What can be observed and what does this mean in a graph? Circle this line in the table.

In the final square, graph the system by solving both equations for y . Also show students how this system can be solved using substitution or elimination. For their System, they can choose which method to use when solving.

Explore (Solving the Task)

In their groups, students are to create a word problem and a system of equations representing the word problem. Before writing on the paper, the word problem and system are to be approved by the teacher.

Focusing Questions

- Are you finding (x,y) for only one line or both lines?
- Should only the x -values match when comparing lines?
- Should only the y -values match when comparing lines?
- Looking at the system, what method makes sense to use when solving? Explain.

Probing Questions

- When do you know the lines have intersected as you look at the table?
- What do the y -intercepts represent?
- How can you explain your solution?
- Do any x -intercepts occur? How would you know by looking at the table? Explain what the x -intercept represents regarding your word problem.

Advancing Questions

- If a group cannot calculate the exact value of the intersection, how can the table indicate between what ordered pairs the intersection occurs?
- What do the lines (points) below the intersection represent?
- What do the lines (points) above the intersection represent?
- If not graphing, looking at the table, what do the ordered pairs below the solution represent?
- If not graphing, looking at the table, what do the ordered pairs above the solution represent?

Elaborate (Discuss Task and Related Mathematical Concepts)

1 – 1 ½ class periods will be needed to complete this task.

The poster will be created in one class period. At that time, each group will write a summary to be attached to their poster including the following:

- 1. What made you use the topic in your word problem? Is it significant to someone in the group?*
- 2. How did you create the 2 linear equations? (Include identifying the variables and constants and the operation.)*
- 3. In your table, did you use only negative or positive numbers? Did you use a combination of negative and positive numbers?*

What x- value does your table begin with and why did you choose to begin with that value?

What does each ordered pair represent? At what ordered pair is the x- and y- values the same? What significance does this particular ordered pair have? What is the vocabulary word used to describe this ordered pair? What is a non-solution to your system?

- 4. What method did you choose to solve your system: Graphing, Substitution or Elimination? Why? What do the lines below and above the intersection represent with regards to your word problem?*

Groups can present their poster and read their summary either to the teacher, other groups or the entire class. Each student in the group is to read a section of the summary. Additional point can be added to the project grade for presenting.

A gallery walk could also be used where students jigsaw the review of the posters.

Checking for Understanding

Purpose: A brief description of what questions or prompts you will use to elicit evidence of student understanding and the strategy you will use to elicit the evidence at the end of the lesson.

Project the following question on the board:

Individually, solve the following system of linear equations using any method of your choosing and answer the questions. Be prepared to share your answers and reasoning with others.

$$y = 6x - 4$$

$$y = (-3)x + 5$$

- 1) Is the solution $x = 1$?
- 2) Why isn't it only $x = 1$?
- 3) What occurs at $(1, 2)$?
- 4) What is a non-solution?
- 5) Which method did you choose and why?
- 6) What is the slope and y-intercept of these lines? This is an example of what formula?

Facilitate a whole class discussion around the system and six questions to ensure student understanding. These questions can also be turned in to True/False questions and a tally of answers for each questions can be kept on the board to enhance discussion.

Common Misunderstanding

Purpose: A brief description of a probe or prompt students could engage in to make them aware of a common misunderstanding

Questions for students:

- 1) In the equation $(-2)y + 5 = 3y - 10$ what is the first step to solve for y ?

Students may want to subtract $2y$ from both sides to have y on the right, however $(-4)y$ will be on the left. A review of opposite values may be helpful and explain what it means to cancel (or eliminate) values.

- 2) How would you solve the following system using the elimination method?

$$x = (-2)y + 5 \text{ and } x = 3y - 10$$

In this example, students may choose to cancel the x which means the equation must first be multiplied by (-1) . Although $x - x = 0$, students ignore this and set $(-5)y = (15)$. Explain that $0 = (-5)y + 15$. Another common mistake when using a multiplier is the constant does not get multiplied. Emphasize that (-1) is multiplied by x , $3y$ AND (-10) .

Closure

Purpose: A brief description of how students will engage in reflecting on their own learning and understanding

Each student will be given $\frac{1}{2}$ a sheet of $8 \frac{1}{2}$ " x 11" paper and asked to draw a triangle in the middle of the paper. (See example in Teachers Material.) At each point of the triangle, the student will write a new 'point' of knowledge they learned during this task.

Extension Task

Purpose: To offer students the opportunity to solve a system of equations using a different method.

Switch posters with another group and solve the solution using the other 2 methods not originally used.

Preferably, select a poster that was not solved by graphing but any method will work. Using the TI-84 Graphing Calculator, solve the problem by graphing. Teach students how to use the TRACE, INTERSECT, and ZOOM feature and view the table. Students will see the ease of solving a system using the TI-84 and finding data at the push of a button.

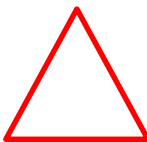
Teacher Materials

Poster Template

Word Problem	System of Equations
Solution and Check	Table

Exit Slip

A system can be solved using different methods.



The solution is shared by both lines

In a table, the solution is where the same x has the same y values.

Summary attachment for poster:

Each group will write a summary to be attached to their poster including the following:

1. *What made you use the topic in your word problem? Is it significant to someone in the group?*
2. *How did you create the 2 linear equations? (Include identifying the variables and constants and the operation.)*
3. *In your table, did you use only negative or positive numbers? Did you use a combination of negative and positive numbers?*

What x- value does your table begin with and why did you choose to begin with that value?

What does each ordered pair represent? At what ordered pair is the x- and y- values the same? What significance does this particular ordered pair have? What is the vocabulary word used to describe this ordered pair? What is a non-solution to your system?

4. *What method did you choose to solve your system: Graphing, Substitution or Elimination? Why? What do the lines below and above the intersection represent with regards to your word problem?*

Checking for Understanding:

Project the following question on the board:

Individually, solve the following system of linear equations using any method of your choosing and answer the questions. Be prepared to share your answers and reasoning with others.

$$y = 6x - 4$$

$$y = (-3)x + 5$$

- 1) Is the solution $x = 1$?
- 2) Why isn't it only $x = 1$?
- 3) What occurs at $(1, 2)$?
- 4) What is a non-solution?
- 5) Which method did you choose and why?
- 6) What is the slope and y-intercept of these lines? This is an example of what formula?

Answer Key:

Project the following question on the board:

Individually, solve the following system of linear equations using any method of your choosing and answer the questions. Be prepared to share your answers and reasoning with others.

$$y = 6x - 4$$

$$y = (-3)x + 5$$

- 1) Is the solution $x = 1$? No, the solution is the ordered pair (1,2) not just $x=1$.
- 2) Why isn't it only $x = 1$? The solution must include an x - and y -value to make both equations true.
- 3) What occurs at (1, 2)? The lines intersect at this point.
- 4) What is a non-solution? A non-solution would be any point other than (1,2). It is a point where the lines do not intersect.
- 5) Which method did you choose and why? Answers will vary.
- 6) What is the slope and y -intercept of these lines? This is an example of what formula? The slope for the top equation is 6 with the y -intercept at (-4). The slope of the bottom equation is (-3) with the y -intercept at 5. Both equations are in the Slope-Intercept Form.

Research and review of standard

Content Standard(s):

Standard(s) for Mathematical Practice:

CCSS.MATH.CONTENT.HSA.REI.C.5
 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

MP 1 – Problem solving
 MP 4 – Model with mathematics

Smarter Balanced Claim

Smarter Balanced Item

Claim 2: Problem Solving
 Claim 4: Modeling and Data Analysis

Next-Generation

20B

Phil and Cath make and sell boomerangs for a school event. They money they raise will go to charity.


They plan to make them in two sizes: small and large.

Phil will carve them from wood. The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve. Phil has a total of 24 hours available for carving.

Cath will decorate them. She only has time to decorate 10 boomerangs of either size.

The small boomerang will make \$8 for charity. The large boomerang will make \$10 for charity. They want to make as much money for charity as they can.

How many small and large boomerangs should they make?
 How much money will they then make?

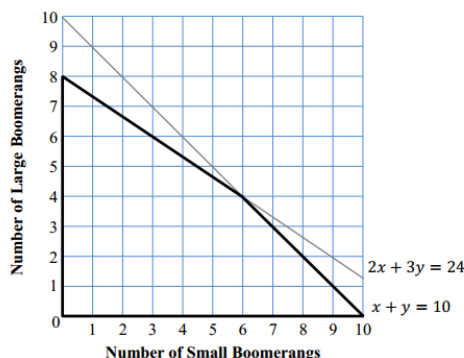


Solution:

DCAS-Like Answer	Next-Generation Solution																													
<p>20A: D (9-11.A.REI.5)</p>	<p>20B: Solutions</p> <p>If one assumes that ten boomerangs are made, then the following table of possibilities may be made. The constraint on carving hours is broken when more than four large boomerangs are made.</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th>Number of Small</th> <th>Profit Made</th> </tr> </thead> <tbody> <tr><td>10</td><td>80</td></tr> <tr><td>9</td><td>82</td></tr> <tr><td>8</td><td>84</td></tr> <tr><td>7</td><td>86</td></tr> <tr><td>6</td><td>88</td></tr> <tr><td>5</td><td>90</td></tr> </tbody> </table> <p>x = small boomerangs y = large boomerangs Time to carve: $2x + 3y = 24$ Can only decorate ten: $x + y = 10$ Solve system of equations</p> <table style="margin-left: 20px;"> <tr> <td>a. $x + y = 10$</td> <td>b. $2x + 3y = 24$</td> <td>If $x + y = 10$</td> </tr> <tr> <td>$y = 10 - x$</td> <td>$2x + 3(10 - x) = 24$</td> <td>$6 + y = 10$</td> </tr> <tr> <td></td> <td>$2x + 30 - 3x = 24$</td> <td>$y = 4$</td> </tr> <tr> <td></td> <td>$-x = -6$</td> <td></td> </tr> <tr> <td></td> <td>$x = 6$</td> <td></td> </tr> </table> <p>Phil and Cath should make 6 small and 4 large boomerangs.</p>	Number of Small	Profit Made	10	80	9	82	8	84	7	86	6	88	5	90	a. $x + y = 10$	b. $2x + 3y = 24$	If $x + y = 10$	$y = 10 - x$	$2x + 3(10 - x) = 24$	$6 + y = 10$		$2x + 30 - 3x = 24$	$y = 4$		$-x = -6$			$x = 6$	
Number of Small	Profit Made																													
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9	82																													
8	84																													
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a. $x + y = 10$	b. $2x + 3y = 24$	If $x + y = 10$																												
$y = 10 - x$	$2x + 3(10 - x) = 24$	$6 + y = 10$																												
	$2x + 30 - 3x = 24$	$y = 4$																												
	$-x = -6$																													
	$x = 6$																													

Next-Generation Solution

This approach, however, does not include the possibility of making fewer than ten boomerangs. A more complete approach would be to draw a graph showing all possibilities.



The possible combinations to be checked are the integer points within the bold region on the graph. The maximum profit occurs, however, when six small and four large boomerangs are made. This profit is \$88. (This can be seen graphically by drawing lines of constant profit on the graph, e.g., $8x + 10y = 80$. This idea may emerge in discussion.)

CPR Pre-Requisites (Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.

Conceptual Understanding and Knowledge

- Understand how to graph an equation written as a linear equation.
- Understand that a solution for a system of equations is a solution for all given equations.
- Understand an expression for y in terms of x is obtained from one of the equations, and should form a true statement when substituted into another equation of the system.
- Identify the solution of a graphed system of equations by observation.

Procedural Skills

- Identify variables.
- Solve for one variable in one equation.
- Substitute the expression representing the variable, into another equation.
- Solve one-step and multi-step equations.
- Substitute an expression for one variable when solving an equation.
- Use labels identifying the variables when constructing an answer.
- Be able to identify the slope and y -intercept when a linear equation is written in slope-intercept form.
- Graph a linear equation on a coordinate plane.
- Observe where the lines intersect to determine a solution.

Representational

- Represent linear relationships as stories, tables, graphs and equations.
- Identify a linear equation when represented in slope-intercept form or standard form.

Social knowledge

- Understand a graphical solution is written as an ordered pair (x,y) .
- Identify $y = mx + b$ is the Slope-Intercept form and know what each variable represents.
Know the names of special solutions (infinitely many, and null set/no solution).

Standards Progression

**Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the “Wiring Document” to help you with this section*

Grade(s) below	Target grade	Grade(s) above
<p><u>CCSS.MATH.CONTENT.6.EE.C.9</u> Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</p> <p><u>CCSS.MATH.CONTENT.7.EE.4a</u> Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</p> <p><u>CCSS.MATH.CONTENT.7.EE.B.4</u> Use variables to represent quantities in a real-world</p>	<p><u>CCSS.MATH.CONTENT.HSA.CED.A.3</u> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.</p> <p><u>CCSS.MATH.CONTENT.HSA.REI.C.6</u> Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p><u>CCSS.MATH.CONTENT.HSA.CED.A.2</u> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p>	<p><u>CCSS.MATH.CONTENT.HSA.CED.A.3</u> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.</p> <p><u>CCSS.MATH.CONTENT.HSA.REI.D.11</u> Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p> <p><u>CCSS.MATH.CONTENT.HSA.REI.D.12</u> Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>

<p>or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p>8.EE.8 Analyze and solve pairs of simultaneous linear equations.</p>		
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Common Misconceptions/Roadblocks
<p>What characteristics of this problem may confuse students?</p> <ul style="list-style-type: none"> • How to separate the information into 2 parts: using the hours to calculate how many boomerangs can be made, and using the dollar values to calculate how much money will be made. • Realizing that a solution can not contain a negative variable, in this context. • Finding values for both variables. • How to interpret the solution indicating the number of small and large boomerangs to make, so the most money can be raised for charity. <p>What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?</p> <ul style="list-style-type: none"> • Students must combine like terms, when possible. • Students do not understand why or how the solution to the system can satisfy all equations. • Students have difficulty identifying and defining the independent and dependent variables. • Students think that they have solved a system after finding the value of only one variable. • Students must understand you can solve for x or y when substituting, then substitute the value of one variable that was solved for, into either of the original equations to find the value of the other variable. <p>What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?</p> <ul style="list-style-type: none"> • Always solve for y, when sometimes solving for x would be easier and more efficient. • Not checking in the original equations.