Mathematics Instructional Cycle Guide

Concept (7.RP.2)

Rosemary Burdick, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Ratio and Proportion

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7.RP.2 Recognize and represent proportional relationships between quantities, fractional quantities, by testing for equivalent ratios in a table or graphing on a coordinate plane.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

Insert the relevant Standard(s) for Mathematical Practice here.
MP.4: Model with mathematics
MP.7: Look for and make use of structure.
MP.8: Look for and express regularity in repeated reasoning.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(p. 21)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(p.3)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(p.7))**
- Supporting lesson materials **(p.20-25)**
- Precursory research and review of standard **7.RP.1 / 7.RP.2** and assessment items that illustrate the standard **(p. 26)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **(Which Cylinder?)** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- Large poster paper for students to work on and then hang around the room.
  - Needed for students to complete their Gallery Walk
- If possible, a smart board

TIME NEEDED

Insert Checkpoint Name: Which Container?
Insert time needed: 20 minutes

Follow-Up Lesson Plan:

Insert time needed: 2-3 Class periods

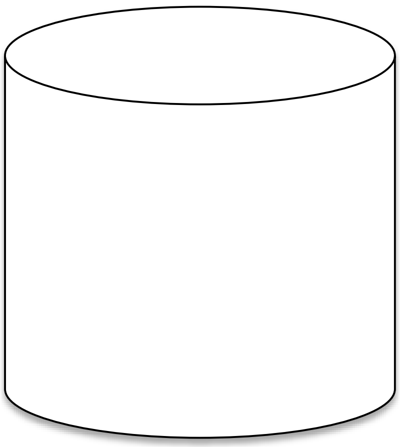
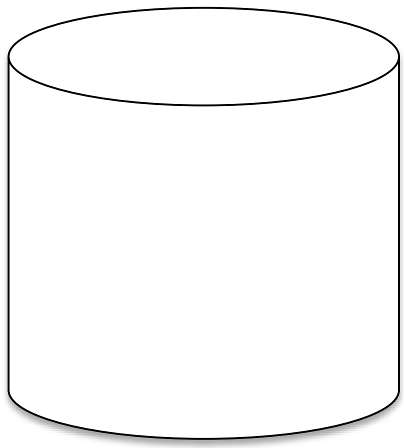
Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">(See attached document on page 18)

Two liquid storage containers of the same size are being filled.

Liquid enters container A at a rate of 2/3 gallon per 1/4 minute.

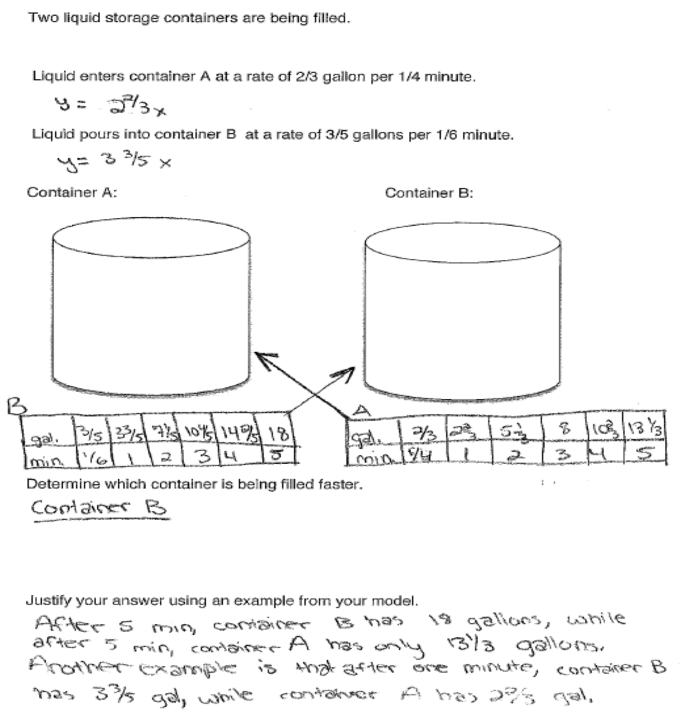
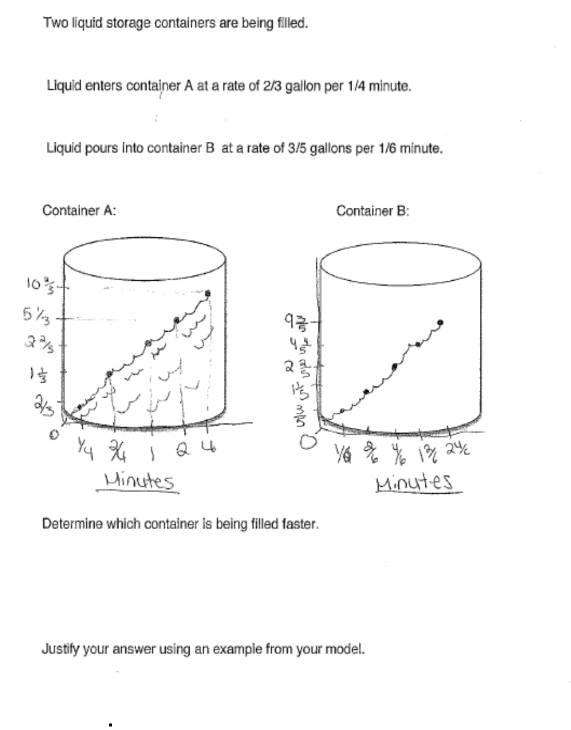
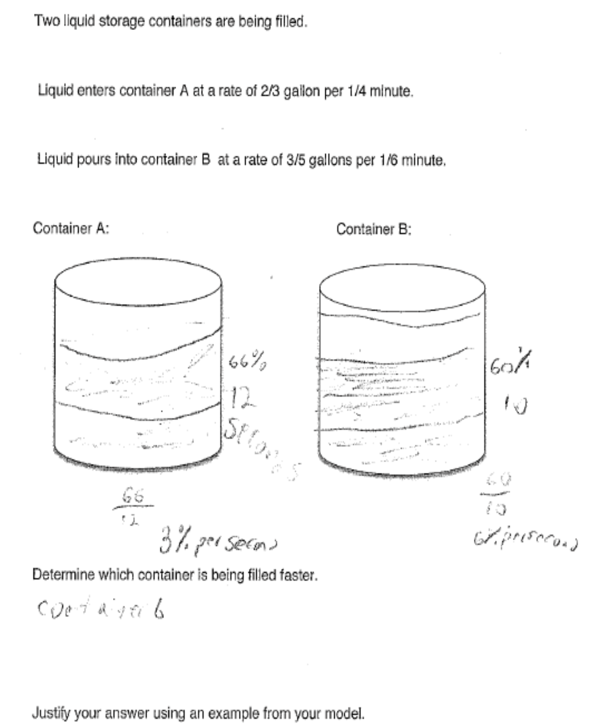
Liquid pours into container B at a rate of 3/5 gallons per 1/6 minute.

Container A:

Container B:


Determine which container is being filled faster.

Justify your answer. Support your reasoning by using evidence from the models above.</td><td>CT Core Standard:</td><td>7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7.RP.2 Recognize and represent proportional relationships between quantities. (Fractional quantities) by testing for equivalent ratios in a table or graphing on a coordinate plane.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>How do students approach a Proportional context involving Fractions, when the unit rate is given as a complex fraction?
To what extent do they?
• Draw a model that correctly represents the division of time and the amount of container filled.
• Connect their model to the situation and how it helped to determine the solution.
• Use the model to make connections to creating a table of equivalent ratios and to writing an equation.
• Solve the equation correctly.</td></tr>
</table>

Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**
What will a response include from a student who has demonstrated conceptual understanding and mastery?  	What will a response include from a student who demonstrated some understanding and possibly some misunderstandings or undeveloped understanding?  	What will a response include from a student who demonstrated minimal understanding and possibly misconceptions? 

Getting Started

Student Response Example

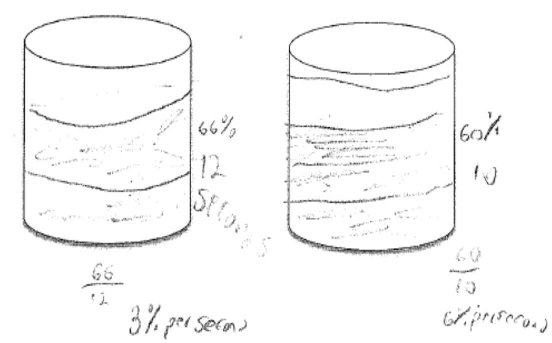
What will a response include from a student who demonstrated minimal understanding and possibly misconceptions?

Two liquid storage containers are being filled.

Liquid enters container A at a rate of 2/3 gallon per 1/4 minute.

Liquid pours into container B at a rate of 3/5 gallons per 1/6 minute.

Container A: Container B:



Determine which container is being filled faster.

container b

Justify your answer using an example from your model.

Indicators

- What possible indicators may be included in a student response who has demonstrated minimal understanding of the standard?
  - Lack of understanding between the relationship of the time and gallons flowing into the two tanks.
  - Lack of a ratio table modeling the relationship between time and gallons.
  - Lack of understanding a unit rate and how that is connected to this situation.
  - The student calculated $\frac{1}{4}$ and $\frac{1}{6}$ of a minute into seconds incorrectly.

- What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
  - No ratio table to show the progression of time and gallons in each container.
  - The student tried to calculate the unit rate by using percentages. He showed 3% of the tank filled per second. That cannot be determined, as the amount of water each tank holds was not given.

- What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
  - Student made no connect between the amount of time and number of gallons flowing into the tanks.
  - Student could not recognize a unit rate.
  - Student did not make the connection between two equal representations of a ratio table.
  - There was not clear understanding that a comparison had to be made comparing the rates to select which container would be filler faster.

In the Moment Questions/Prompts

What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking?

- What relationship is represented in this problem
- Looking at Container A – how much water is flowing into the container and in what time period.
- Let's break down the time periods and see how much water will be in the tank at each time:

$\frac{1}{4}$ hour – how much water ______________________________

A second $\frac{1}{4}$ hour – how much water ____________________________

After a third $\frac{1}{4}$ hour – how much water ________________________

With this students I would draw the bar model to have them fill in with time – and then the gallons that entered the tank after each $\frac{1}{4}$ hour.

I would ask the student how many $\frac{1}{4}$ hours are in 1 whole hour.

Then I would ask for them to mark off that time on a Bar model – and see if they can determine the amount of water in the tank.

$\frac{1}{4}$ *hour*	$\frac{1}{4}$ *hour*	$\frac{1}{4}$ *hour*	$\frac{1}{4}$ *hour*

If a student continues to have a hard time with the concept – I would then move to model using half hours – or ONLY whole numbers to see if the student can then see the relationship between the numbers.

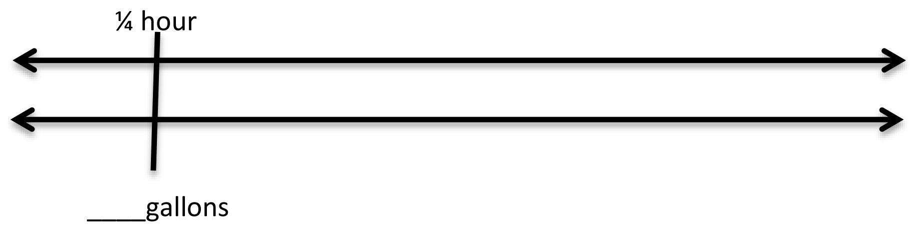
Closing the Loop (Interventions/Extensions)

LZ video lesson links that may help develop conceptual understanding and procedural skill needed

http://learnzillion.com/courses/43?collection_id=552

I also use with the student a visual model showing the relationship. This would be found at: http://www.thinkingblocks.com

Here students will visually see the connection and relationship between numbers. After the student worked through the guided lessons here, I would use Thinking blocks to model this problem

<table>
<tr>
<td>A double number line could also be used to help the students the relationship between time and gallons.

</td>
<td></td>
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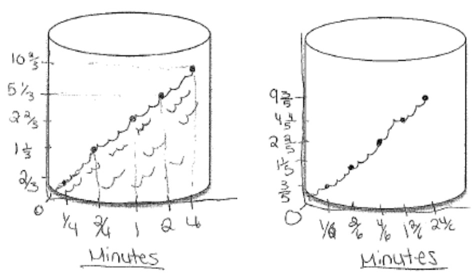
</table>

<table>
<tr><th colspan="2">Developing</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr>
<td>
What will a response include from a student who demonstrated some understanding and possibly some misunderstandings or undeveloped understanding?

Two liquid storage containers are being filled.

Liquid enters container A at a rate of 2/3 gallon per 1/4 minute.

Liquid pours into container B at a rate of 3/5 gallons per 1/6 minute.

Container A: Container B:


Determine which container is being filled faster.

Justify your answer using an example from your model.
</td>
<td>

What possible indicators may be included in a student response who has demonstrated some understanding of the standard?

Student showed no ratio table for the relationship between time and gallons.
Student did not write the unit rate for each tank.
No equation was given to show the relationship of this situation.

What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?

Student did understand that the filling of the containers was in a linear relation – they show the water-level moving in a straight line.

What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?

For Contain A the student showed a doubling of time increments. He was not consistent with his measurements.
The container that would fill the fastest was not indicated.
Student did not show recognition of unit rate.

</td>
</tr>
</table>

In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking? Your idea of drawing water, was an awesome idea. Can you talk to me about your model? What is it showing and how do you feel it models the situation given in the question? Can you explain your increments of time for container A? Why did you not use the same idea for container B? Do you think you could have used another type of model to help you show how much water was going into the tank for each time period?	LZ video lesson links that may help develop conceptual understanding and procedural skill needed http://learnzillion.com/courses/43?collection_id=552

<table>
<tr><th colspan="2">Got it</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td>What will a response include from a student who has demonstrated conceptual understanding and mastery?</td><td>

What indicators must be included in an exemplar student response Students how understood this question would have shown:

A clear understanding of ratio table
Given the correct equivalent fractional ratios.
Use of division with complex fractions.
Understanding of unit rate and how it is used in an equation.

What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?

The student has the ability to:

Describe and identify complex fractions
Recognize the difference between a unit rate and a ratio
Recognize that a unite rate can be fractional
Recognize that two equivalent ratios represent a proportion.
Recognize the connections between the equivalent ratios and the values in the table.
Recognize the unit rate as the constant of proportionality
Use the constant of proportionality in an equation.

What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?

A student may not make a connection between the constant of proportionality and the unit rate, or how that is used in an equation.

</td></tr>
</table>

Two liquid storage containers are being filled.

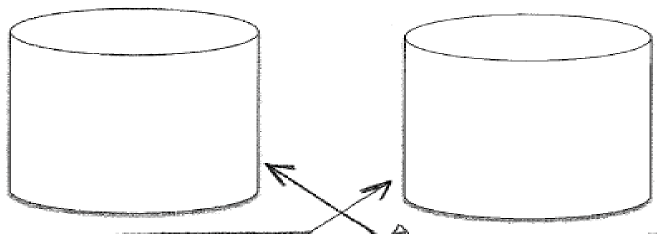
Liquid enters container A at a rate of 2/3 gallon per 1/4 minute.

$y = 2\frac{2}{3}x$

Liquid pours into container B at a rate of 3/5 gallons per 1/6 minute.

$y = 3\frac{3}{5}x$

Container A: Container B:



B

gal.	3/5	3 3/5	7 1/5	10 4/5	14 2/5	18
min	1/6	1	2	3	4	5

A

gal.	2/3	2 2/3	5 1/3	8	10 2/3	13 1/3
min	1/4	1	2	3	4	5

Determine which container is being filled faster.

Container B

Justify your answer using an example from your model.

After 5 min, container B has 18 gallons, while after 5 min, container A has only 13 1/3 gallons. Another example is that after one minute, container B has 3 3/5 gal, while container A has 2 2/3 gal.

In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
What questions could you ask, or feedback could you provide in the moment to extend or push student understanding, create disequilibrium, or advance student thinking? I would ask that student how they felt the problem would change if I told them each container held 35 gallons or 35 $\frac{3}{5}$ gallons. How do you think a picture of this ratio table would look? If I wanted to graph it what do you think I should do?	LZ video lesson links that may extend and deepen student understanding and procedural fluency **http://learnzillion.com/lessons/2966-calculate-a-unit-rate-with-two-fractions-using-division** **http://learnzillion.com/lessons/3061-analyze-a-situation-using-a-rate-table** **http://learnzillion.com/courses/43?collection_id=552**

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	**Modeling proportional relationships and unit rate with fractional units.**
Content Standard(s):	**7.RP.2 Recognize and represent proportional relationships between quantities, (Fractional quantities) by testing for equivalent ratios in a table or graphing on a coordinate plane.**
Targeted Practice Standard :	**MP.4:** Model with mathematics. **MP.7:** Look for and make use of structure. **MP.8:** Look for and express regularity in repeated reasoning.

Mathematical Goals	Success Criteria
Students will move from the Bar Model approach (6th grade) to understanding a proportional representation with fractional units – and learn how to the use a ratio table to find unit rate.	*Students need to understand, recognize and utilize proportional relationship in tables, graphs and equations. This lesson is to help the students move from the Bar Model – introduced in 6th grade – to the ratio table to the determining the unit rate.*

Launch (Probe and Build Background Knowledge)

Prior Knowledge: Students should understand the concept of ratio and use precise language and symbols to describe a ratio relationship. Students should also understand unit rate and use ratio and rate reasoning to solve mathematical problems

Purpose: Asses and Activate Prior Knowledge:

Students will compute unit rate associated to fractional quantities, which measure different units to find which water tank will be filled the fastest.

The student's **Do Now** would be:

Without solving compare the two problems below. Use the questions provided to help you compare them.

1) Oliver is training for a marathon. In practice he runs 15 kilometers in 70 minutes. How far will Oliver run in 1 minute?

2) On the snowiest day in the past 5 years, it snowed 7 feet in $27\frac{1}{2}$ hours. How much did it snow per hour?

- How are these two problems the same? How are they different?
- What can we learn from doing problem #1 that will help us solve problem #2.

Instructional Task

Purpose: *A brief description of the mathematics and/or the mathematical practices the task is intended to engage students in and what students will be doing.*

Students will refine their skills from the 6th grade curriculum, transferring them to problem solving to find unit rates when given fraction/decimal quantities.

a) Students will demonstrate the solutions to finding unit rates when given two quantities including whole numbers, fractions, and decimals.

b) Students find unit rates within real world situations using whole and non-whole number quantities. (Examples: finding the rate of speed distance/time, cost per unit). Students can be given scenarios where given different distances and times, they compare rates/speeds. Faced with two retail items students can determine the unit price and reason which is the better deal.

Barry wants to enter a local bike race. He begins his training by biking 8 1/2 miles every 1/2 hour.

What are the two units be compared in this problem?

Create a model to determine how far Barry will bike in 2 1/2 hours.

If this rate continues, how long will Barry have to ride, to reach his goal of 51 miles?

Using the model you created, write an equation to model this situation for any number of miles"

Engage

(Setting Up the Task)

Using the Bike problem, above on page 13, ask the students to work in groups and discuss how they will show the distance vs. the time ratio. Check for prior knowledge and use of the Bar Model to develop a "picture" of the time distance relationship. Students will post their work on large paper around the room and share these ideas. A Gallery Walk will then be completed by the other students who will comment on the work and write any questions they may have. Students will then share their work and answer any of the questions posted on their paper.

Explore (Solving the Task)
Present the Container Problem.

- *After a brief introduction to the problem, have the students work in groups – and be prepared to place their work on large paper to hang up around the room.*

What questions will you ask as students work on the task to elicit evidence of their understanding and support mathematical connections?

Focusing Questions

- What information do you know that will help you with this question?
- What type of model will you be using to show that information?
- What is a unit rate?

Probing Questions

- What decision helped you determine what model to use?
- How does your model relate to division/multiplication of fractions?
- How does your model relate back to each container?

Advancing Questions

- Is there any other way you could have answered this question?
- How would we change this question to determine which container would be filled first?

- How can we find a unit rate?
- What connection is there to the unit rate and your model?
- What connection can you make between the unit rate and the equation?
- How does your model show the solution to the question?
- Have your checked your solution?

What are some anticipated student responses or solution paths?

- *How do we know which will fill faster – we do not know how much water each container will hold?*
- *Give students some direction to the type of model that can use: Box model / Bar model / Double line graph / ratio table.*
- *If students are having a huge problem working with the fractions, ask them to make the problem simpler and work with whole numbers first. Then move to fractions that might be easier for them to work with.*

Elaborate (Discuss Task and Related Mathematical Concepts)

How will you facilitate the sharing of student work and discussion to support students in making mathematical connections?

- *Students' work will be done on large paper and hung around the classroom.*
- *Students will then complete a Gallery Walk.*
  - *They must give one positive feedback.*
  - *They ask a question about the work?*
    - *Did you think about using a double line chart instead of _______model?*
    - *Why did you set up your ratio table as you did?*
- *Each group will have an opportunity to share their work – and answer any question(s) place on their paper.*

Call the class back together to discuss their observations.

Refer back to the Question presented before this lessons – How can we model proportional relationships when our unit rates are fractions?

As students share their answer to this question their responses will be written on the white board.

Finally come to a class agreement on a response.

Checking for Understanding

Purpose: *Pose the following as an exit card to elicit evidence of students' understanding of modeling proportional relationship using fractional unit rates. (See Attached on page 20)*

Exit Ticket:

Name 2 concepts you learned today.

What question(s) do you still have with today's lesson?

Can you help Ms. Alberto's students?

Ms. Alberto decided to make lemonade to serve to her math club students. The directions said to mix 2 scoops to powered drink mix to ½ gallon of water to make each pitcher of lemonade. One of Ms. Alberto students tells her she will need to add 8 scoops of powered mix with 2 gallons of water so she can make 4 pitchers of lemonade. How can you use the concept of today's lesson- ratio tables - to tell whether this student is correct?

Common Misunderstanding

Purpose: *To address common misunderstanding students often have about fractions and how they relate to today's lesson remind the students of the task used in the Do Now.*

1) Oliver is training for a marathon. In practice he runs 15 kilometers in 70 minutes. How far will Oliver run in 1 minute?

2) On the snowiest day in the past 5 years, it snowed 7 feet in $27\frac{1}{2}$ hours. How much did it snow per hour?

- How are these two problems the same? How are they different?
- What can we learn from doing problem #1 that will help us solve problem #2.

Depending on their concern:

- Remind students of the connection between ratios and unit rates.
  - How are rates and ratio alike? How are they different

If the student has not developed for themselves a personal knowledge / understanding of the conceptual models used in division of fractions. Explore ways that it can be done.

- What does unit rate mean? Can you think of ways you have heard it used in your daily life?
- In question one how did you find the unit rate? Do you think that process would change in the second question?

If the students are having a problem creating the ratio table, ask the following:

- Do you remember how we found equivalent fractions? Can you describe that process ~~to me~~?
- How can we use that process to find equivalent ratios?

Help the students to make a connection with the unit rate and how it progress throughout the ratio table

- As you moved from ¼ minute to ½ a minute – what did you notice about what you were doing to the time? Therefore what must you do with the amount of water flowing into the tanks?
- Do you think there is a connection between your unit rate and the rest of the table?
- How might this unit rate help me solve this problem?
- How can this unit rate be used in an equation?

Checking for Understanding

Purpose: *A brief description of what questions or prompts you will use to elicit evidence of student understanding and the strategy you will use to elicit the evidence at the end of the lesson.*

Using the same basic question of which container is filling faster ~~–~~ask the following. Students would still have time to work with their partner – they would do a THINK – PAIR – SHARE with the rest of the class.

What would happen if I told you Container A would hold 50 gallons of water and Container B would hold 65 gallons of water? Would that change your answer to the question, which container is filling faster?

If both of the Containers would hold 75 gallons of water – which container would fill first and why?

Can you explain the difference between filling faster and which is filled first? What information do you have now that would determine this?

Closure

Purpose: *Before students are given the exit card provide an opportunity for them to answer the Essential question for themselves in the following I can statements:*

	With no problem!!	I still have a little concern.	Need help
I can draw a model or use a ratio table to help me determine equivalent fractional ratios?			
I can use the unit rate to help me determine the relationship in my model?			
I can use the unit rate in an equation.			
I can justify my answer through using examples from my model?			

Were you are creative problem solver today?

Explain how.

Extension Task

Purpose: *Provide an extension task for those students who are ready to deepen their understanding of proportional relationships and fractional rates.*

A local store makes a special citrus salad dressing to be sold in their stores and used on their salads. Here is a listing of ingredients.

$\frac{1}{3}$ of the mixture is olive oil
$\frac{1}{5}$ of the mixture is balsamic vinegar
$\frac{1}{4}$ of the mixture is orange juice
$\frac{1}{4}$ of the mixture is lemon juice

The store wants to make sure that very batch is kept in the same proportions.

	Single Batch	**Double Batch**	**Triple Batch**	**Quadruple Batch**	**A batch that is X size of the original**
Olive Oil					
Balsamic vinegar					
Orange Juice					
Lemon juice					

Does the process change now since you are making a more extensive ratio table?

How are you finding the amount of ingredients for the Batches?

How do you use your unit rate in this process?

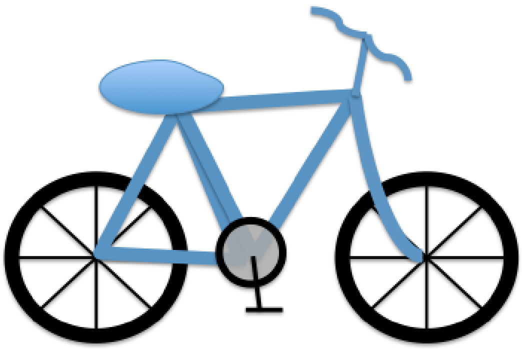
Suppose the number of Batches I wanted to make were not on your ratio table? Explain how you would determine how much of each ingredient I would use to make that batch?

Use your ideas listed about, and determine how much of each ingredient I would need to make 12 batches.

Were your ideas in line with what you did? If so explain how?

Name: ________________________________ Date: ______________

Bike Problem



Barry wants to enter a local bike race. He begins his training by biking 8 1/2 miles every 1/2   hour.

What are the two units be compared in this problem?

Create a model to determine how far Barry will bike in 2 1/2 hours.

If this rate continues, how long will Barry have to ride, to reach his goal of 51 miles?

Using the model you created, write an equation to model this situation for any number of miles.

Name: ______________________________Date:_____________

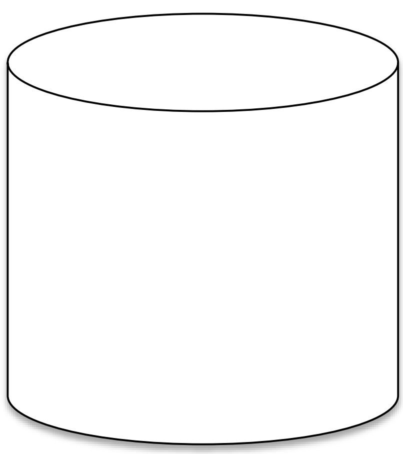
Which Container?

Two liquid storage containers of the same size are being filled.

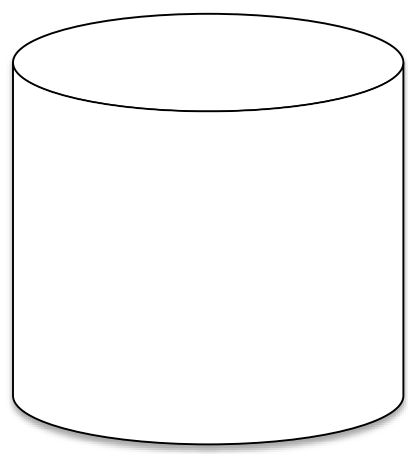
Liquid enters container A at a rate of $\frac{2}{3}$ *gallon* per $\frac{1}{4}$ *minute*.

Liquid pours into container B at a rate of $\frac{3}{5}$ gallon per $\frac{1}{6}$ *minute*.

Container A:



Container B:



Determine which container is being filled faster.

Justify your answer. Support your reasoning by using evidence from the models above.

Name: ______________________________ Date: ________________

Special Salad Dressing

A local store makes a special citrus salad dressing to be sold in their stores and used on their salads. Here is a listing of ingredients.

- $\frac{1}{3}$ of the mixture is olive oil
- $\frac{1}{5}$ of the mixture is balsamic vinegar
- $\frac{1}{4}$ of the mixture is orange juice
- $\frac{1}{4}$ of the mixture is lemon juice

The store wants to make sure that very batch is kept in the same proportions.

	Single Batch	**Double Batch**	**Triple Batch**	**Quadruple Batch**	**Batch that is x time greater.**
Olive Oil					
Balsamic vinegar					
Orange Juice					
Lemon juice					

Does the process change now since you are making a more extensive ratio table?

How are you finding the amount of ingredients for the Batches?

How do you use your unit rate in this process?

Suppose the number of Batches I wanted to make were not on your ratio table? Explain how you would determine how much of each ingredient I would use to make that batch?

Use your ideas listed about, and determine how much of each ingredient I would need to make 12 batches.

Were your ideas in line with what you did? If so explain how?

Name: ____

Exit Ticket:

Name 2 concepts you learned today.

What question(s) do you still have with today's lesson?

Can you help Ms. Alberto's students?

Ms. Alberto decided to make lemonade to serve to her math club students. The directions said to mix 2 scoops to powered drink mix to ½ gallon of water to make each pitcher of lemonade. One of Ms. Alberto students tells her she will need to add 8 scoops of powered mix with 2 gallons of water so she can make 4 pitchers of lemonade. How can you use the concept of today's lesson - ratio tables - to tell whether this student is correct?

Name: ____

Exit Ticket:

Name 2 concepts you learned today.

What question(s) do you still have with today's lesson?

Can you help Ms. Alberto's students?

Ms. Alberto decided to make lemonade to serve to her math club students. The directions said to mix 2 scoops to powered drink mix to ½ gallon of water to make each pitcher of lemonade. One of Ms. Alberto students tells her she will need to add 8 scoops of powered mix with 2 gallons of water so she can make 4 pitchers of lemonade. How can you use the concept of today's lesson - ratio tables - to tell whether this student is correct?

Name: ____

Self-Reflection:

This will be placed in your portfolio, stapled to today's container question.

	With no problem!	I still have a little concern.	Need help
I can draw a model or use a ratio table to help me determine equivalent fractional ratios?			
I can use the unit rate to help me determine the relationship in my model?			
I can use the unit rate in an equation.			
I can justify my answer through using examples from my model?			

Explain how to using a ratio table helped you with today's question?

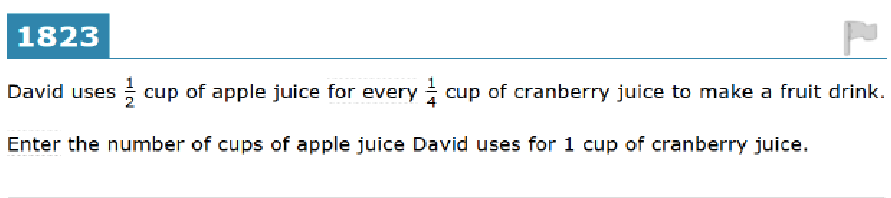
Were you a creative problem solver today?

Explain how.

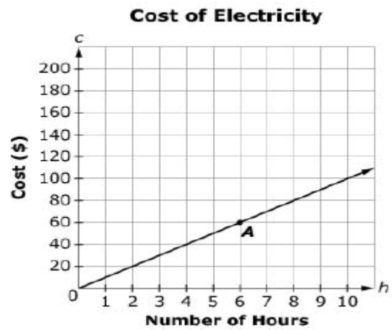
Scenario:

Table:

Equation/Algebraic Model:

Research and review of standard	
Content Standard(s):	**Standard(s) for Mathematical Practice:**
What standard was this item designed to assess? **Ratio and Proportion** **7.RP.1** Compute unit rates associated with ratios of fraction, including ratios of length, areas and other quantities measured in like or different units. **(COMPLEX Fractions)** **7.RP.**2 Recognize and represent proportional relationships between quantities. (Fractional quantities) by testing for equivalent ratios in a table or graphing on a coordinate plane.	*What Standard(s) for Mathematical Practice are implicit in this item or content standard?* **MP.2:** Reason abstractly **MP.3:** Construct Viable arguments and critique the reasoning of others. **MP.4:** Model with mathematics. **MP.7:** Look for and make use of structure.
Smarter Balanced Claim	***Smarter Balanced Item***
Claim 1: *Concepts and Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedure with precision and fluency.*	*Select a Smarter Balanced released item that addresses your selected standard(s). If possible, insert a screenshot here.*  1823 David uses $\frac{1}{2}$ cup of apple juice for every $\frac{1}{4}$ cup of cranberry juice to make a fruit drink. Enter the number of cups of apple juice David uses for 1 cup of cranberry juice.

This graph shows a proportional relationship between the number of hours (h) a business operates and the total cost (c) of electricity.



Select True or False for each statement about the graph.

	True	False
Point *A* represents the total cost of electricity when operating the business for 6 hours.	☐	☐
The total cost of electricity is $8 when operating the business for 80 hours.	☐	☐
The total cost of electricity is $10 when operating the business for 1 hour.	☐	☐

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<td>CPR Pre-Requisites (Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.</td>
<td>7.RP.1 is extensions of the students work in sixth grade on division of fractions. In sixth grade students had to use the Bar Model, and Double number lines to gain an understanding of division of fractions. 7.RP.1 is a combination of division of fractions moving into unit rate. 7.RP.2 students will begin to use this concept in the creation of ratio tables – another model to aid them in understanding ratios. It is a full understanding of the unit rate that will then help the students move on to the understanding of the equation.
Conceptual Understanding and Knowledge

What are the conceptual understandings students must have in order to achieve mastery of the standard

Clear understanding of what a fraction is.
Remainder is a portion of the group – "serving size" the remainder is a piece of what we are looking for.
Connection between ratio and fractions and their differences.
Clear distinction between multiplying and dividing fractions.
Example: 8 x 4 same as 8 $\div \frac{1}{4}$
That a Complex fraction is another way to represent a basic fraction division problem
$\frac{1}{3} \div \frac{1}{2}$ is the same as $\frac{\frac{1}{3}}{\frac{1}{2}}$

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	- Ratios are often expressed in fractional notation, although ratios and fractions do not have identical meaning.
	- Ratios are often used to make "part-to-part" comparisons; fractions are only a "part-to-whole" comparison.
	Procedural Skills
	• What are the pre-requisite procedural skills and strategic competencies students must have in order to achieve mastery of the standard
	  - Multiplying Fractions
	  - Division of Fractions
	  - Understand the concept of unit rate
	  - Recognize the difference between unit rate and a ratio
	  - Recognize that a unit rate can be fractional
	  - The ability to recognize and represent the connection between equivalent ratios and values in a table
	  - The ability to determine the unit rate as the constant of proportionality
	  - The ability to use the constant of proportionality in an equation
	
	Representational
	• What representations should students be able to understand and use in order to achieve mastery of the standard
	  - Use of Bar Model for division of fractions
	  - Use of Number line to understand division of measurement. (Time and distance)
	  - Double Number line for complex fractions
	It is the use of these representational models that will help us lead students to the ratio table and the understanding of unit rate.
	  - Equivalent ratio table
	  - Equation to represent the relationship found in the ratio table.
	Social knowledge
	• What terms, definitions, and conventions must students have knowledge of in order to achieve mastery of the standard
	  - Numerator / denominator / part-to-whole ratio / complex fractions.
	  - Distance can be represented on a number line.

Standards Progression **Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*		
Grade(s) below	**Target grade**	**Grade(s) above**
What previous grade level standards build up to the grade level standard this item assesses? **Numbers and Operations: Fractions** **5.NF.3** Interpret a fraction as division of the number by the denominator. **5.NF.4 a, b** Extend and Apply previous understanding of multiplication to multiply a fraction or whole number by a fraction. **5.NF.7:** Apply and extend previous understanding of division to divide unit fractions by whole numbers and whole numbers by a unit fraction. **Number System:** **6.NS.1:** Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions.	*What other grade level standards are connected to the standard this item assesses?* **Number System:** **7.NS.2:** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **7.NS.2c:** Apply properties of operations as strategies to multiply and divide rational numbers. 7.RP.2c: Represent proportional relationships by equations 7.RP. 3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*	*What subsequent grade level standards build off of the grade level standard this item assesses?* **Number System:** **8.NS.1:** Know that numbers that are rational are called irrational. **8.NS.2 Use** rational approximations of irrational numbers to compare the size of irrational numbers 8.F.1: Functions

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- Students lack a general conceptual understanding of fractions
- Student lack a general conceptual understanding of ratios and unit rates.
- Students have not developed for themselves a personal knowledge / understanding of the conceptual models used in division of fractions.
- Student might see the numerator and denominators as whole numbers and not as a unit by themselves.
- Students do not understand a fraction represents a number between 0 and 1.
- Students will not understand how a fraction in itself can be a numerator or a denominator.
- Division of fractions
    - *Will division always make things smaller?*
    - *Will multiplication always make things bigger?*
    - *When will this be true? When will it NOT be true?*
- Students have not mastered the concept of relationships between quantities or how equivalent ratios are built.

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students may not fully understand a fraction is less than 1.
- Fractions in this question can easily add up to 1 minutes, which then becomes the unit rate.
- Addition of fractions with like denominators could also be a misconception for students who do not fully understand they add only adding numerators.
- The relationship between a complex fraction $\frac{\frac{2}{3}}{\frac{1}{4}}$ and the basic algorithm of $\frac{2}{3} \div \frac{1}{4}$.
- For some students with a misconception of division of fractions, it may become necessary to take the example back to the beginning of whole numbers divided by 1 – such as $8 \div 1 =$ _____. If we divide a number by 1 what happens? How do you think this will change if we divided 8 by a number less than one?

What over generalizations may students make from previous learning leading them to make false connections or conclusions?

- That division will also make things smaller.
- That a ratio is just another way to name a fraction.
- Meaning of a fraction with a denominator of 1 - How that relates to unit rate.
- That the unit rate for a relationship such as ¼:1/2 would be less than 1 since both ratios are less than 1.