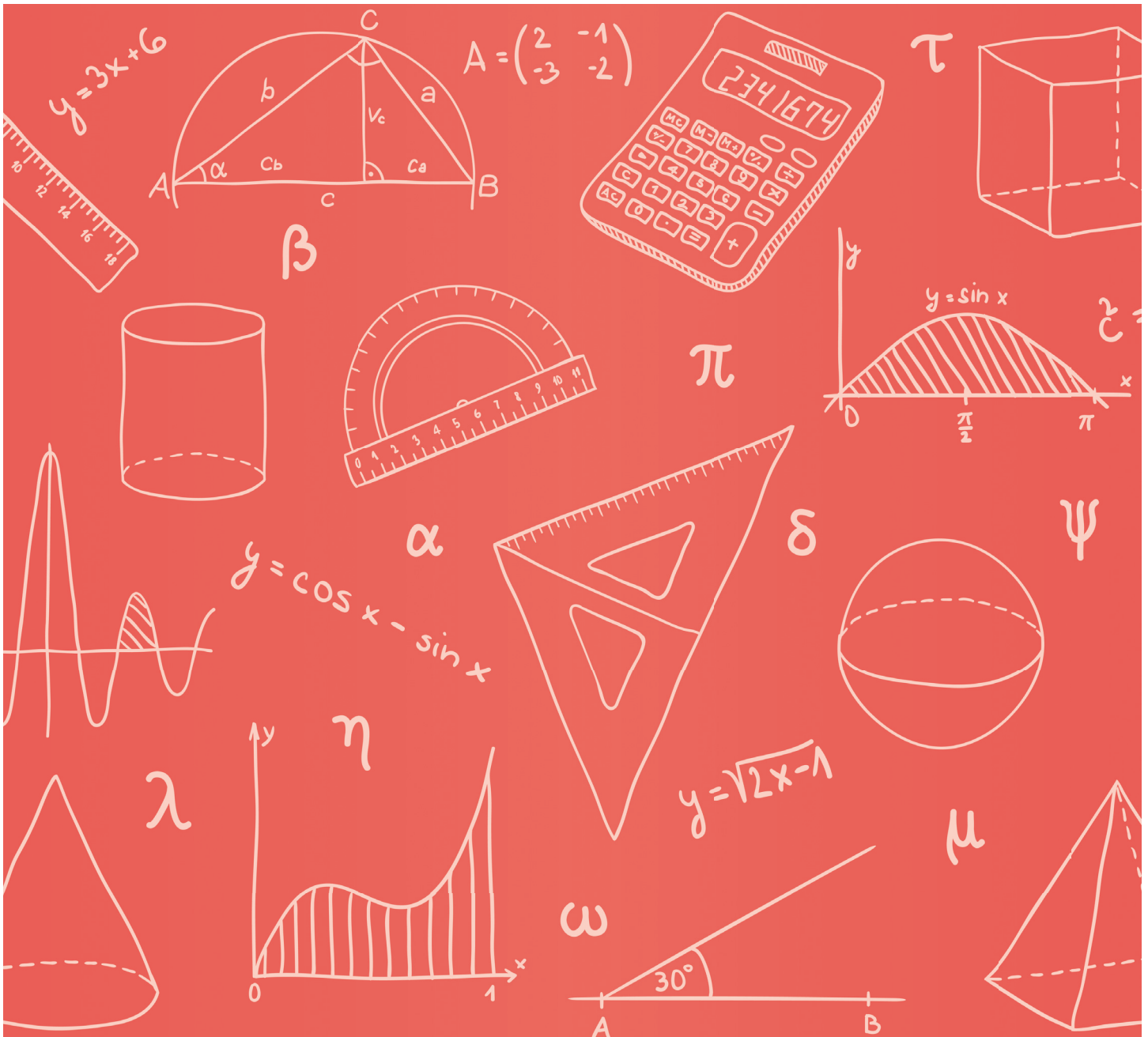


LESSONS FOR LEARNING

FOR THE COMMON CORE STATE STANDARDS IN MATHEMATICS



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M0910

Seventh Grade – Standards

1. Developing understanding of and applying proportional relationships –

Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Developing understanding of operations with rational numbers and working with expressions and linear equations – Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes

to solve problems involving area, surface area, and volume – Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Drawing inferences about populations based on samples – Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

RATIOS AND PROPORTIONAL RELATIONSHIPS

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- 7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $(1/2)/(1/4)$ miles per hour, equivalently 2 miles per hour.*
- 7.RP.2** Recognize and represent proportional relationships between quantities.
- a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
 - c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.*
 - d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
- 7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

THE NUMBER SYSTEM

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

- 7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

- b. Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
 - c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
 - d. Apply properties of operations as strategies to add and subtract rational numbers.
- 7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
 - b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-p/q = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
 - c. Apply properties of operations as strategies to multiply and divide rational numbers.
 - d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. (NOTE: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

EXPRESSIONS AND EQUATIONS

Use properties of operations to generate equivalent expressions.

- 7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

GEOMETRY

Draw, construct, and describe geometrical figures and describe the relationships between them.

- 7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- 7.G.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

- 7.G.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- 7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STATISTICS AND PROBABILITY

Use random sampling to draw inferences about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

James Bond

Common Core Standard:

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Additional/Supporting Standard(s): NA

Standards for Mathematical Practice:

1. Model with mathematics
2. Use appropriate tools strategically
3. Attend to precision

Student Outcomes:

- I can add integers

Materials:

- James Bond gameboard for each pair of students
- Game markers

Advance Preparation:

- Review adding integers in relation to moving up and down the number line
- Prepare the game boards and spinner
- An easy way to make spinners is to punch a paper brad through the center of a spinner, and place a large paper clip over the brad. To improve this technique, cut off a small section of straw to pass the brad through before you punch it through the paper. The straw forms a smooth wall for the paper clip to spin around.

Directions:

1. Pair students to play the game.
2. Give each pair a game board with the spinner put together and game markers
3. Students start at sea level or 0 and take turns spinning. They add the number they spin to the number at their location.
4. Players must move all the way to the bottom of the ocean and return to the top of the cliff in order to win
5. Players must land exactly at the ocean floor or the cliff. When a player lands on the same location as his opponent (directly across), the player moves up ten spaces and the opponent moves down ten spaces

Questions to Pose:Before:

- What do you do if you are ten spaces from the ocean floor and you spin -20?
- Where is sea level located on the gameboard?

After:

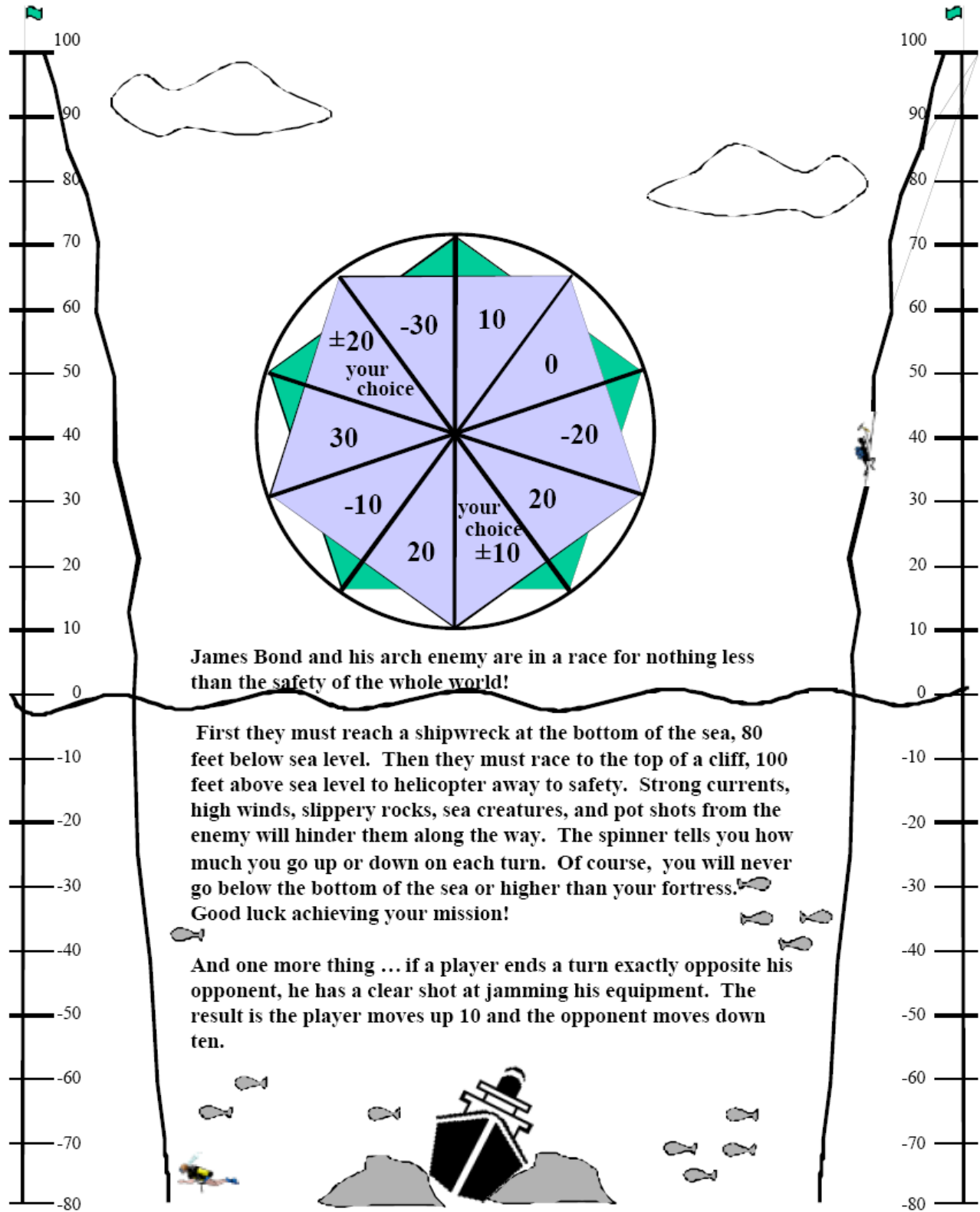
- Why does the game get more difficult when you get close to the ocean floor or the top of the cliff?
- What did you do to decide the number you had to spin in order to win the game?

Possible Misconceptions/Suggestions: NA**Special Notes:**

Numbers on the gameboard and spinner can be changed to make the practice more difficult and to include fractions and decimals.

Solutions: NA

James Bond Game



“Sign” Your Name

Common Core Standard:

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Additional/Supporting Standard(s): N/A

Standards for Mathematical Practice:

6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

Student Outcomes:

- I can add integers.
- I can determine the absolute value of a quantity.

Materials:

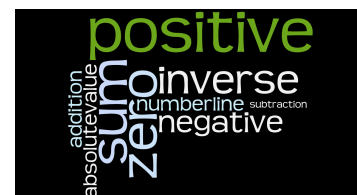
- “Sign” Your Name handout
- Internet access to create a class Wordle of student names; <http://www.wordle.net/>

Advance Preparation:

- Students should be familiar with signed numbers and how to use a number line to help with signed addition.
- Students need to have an understanding that absolute value is the distance from zero on a number line.

Directions:

1. Show students the integer Wordle and discuss why some words appear larger and others appear smaller. Tell them that the activity today will allow them to create a class Wordle and that we will mathematically determine the size of our names in the Wordle. The Integer Wordle can be found at the following link: <http://www.wordle.net/show/wrdl/5512350/Integers>
2. Provide students a copy of the “Sign” Your Name handout.
3. Students should complete each question of the task to practice using a number line when adding integers.



4. At the end of the lesson allow students to design a Wordle on the computer with the names of all students in the class. (<http://www.wordle.net/>) Use the absolute value of each student's first name. Have each student type their first name in the Wordle the number of times that equals the absolute value of their name. Print out the class Wordle and display.

Example: JULIE = $-3 + 8 + (-1) + (-4) + (-8) = -8$; $|-8| = 8$

DAN = $-9 + (-12) + 1 = -20$; $|-20| = 20$

ALISAN = $-12 + (-1) + (-4) + 6 + (-12) + 1 = -22$; $|-22| = 22$

NANCY = $1 + (-12) + 1 + (-10) + 12 = -8$; $|-8| = 8$

Julie will type her name 8 times in the Wordle program. Dan will type his name 20 times, Alisan 22 times and Nancy 8 times. The student whose name has the largest absolute value will appear the largest in the Wordle. The student whose name has the smallest absolute value will appear the smallest in the Wordle.

5. Now have the students create a Wordle that will display the true value of their first name. Student names that have negative values will be typed in backwards to represent the additive inverse value. Since we cannot type a name in Wordle a negative amount of times, the issue of negatives will be addressed by adding one more than the absolute value of the smallest valued name.

Using the example above, ALISAN has the smallest valued name at -22. The absolute value of -22 is 22 then add one more to obtain a new value of 23. Adding 23 to each student's first name value will ensure that the student with the lowest name value will appear as the smallest in the Wordle which will be equal to 1. This same rule will now be applied to all students in the class. Thus, JULIE now has a value of $-8 + 23$ or 15; DAN will be $-20 + 23$ or 3; ALISAN is now $-22 + 23$ or 1, and NANCY is now $-8 + 23$ or 15. The amount added to each student's name value will depend on the smallest value in each class. The end result should be that the student with the lowest name value will enter their name in the Wordle one time.

Use the same process as in the previous Wordle by having students type their name in the Wordle program with their new value. A cool twist is to have the students whose first name was originally negative (before adding 23 as in our example), type their name in backwards so that it will be clear on the Wordle that their name value was in fact negative.

Questions to Pose:

Before:

- Can you predict which student's name in our class will have the highest value when we apply the given code? Can you predict who will have the lowest valued name?
- What is your reasoning for your predictions?

During:

- What patterns did you notice when adding integers on the number line?
- Can we make some general rules for adding integers, those with like signs and those with different signs?
- Would the order of the values in a name matter when finding the total?

After:

- How does your name size on the absolute value Wordle compare to your name size on the adjusted true value Wordle?
- What is the reasoning for the change in your name size?
- What is the reasoning for some names being typed in backwards?

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Students often misunderstand the value of negative numbers. For example, students often state that $-1 < -10$, as if the numbers were positive.	Review with students that when comparing two positive integers, the number further to the right on the number line is always larger. The same reasoning applies to negative numbers on the number line. The larger value will always be the one further to the right.

Special Notes:

Some student names may require movement or result in a sum larger than the length of the provided number line. Based on the need of your students, a longer number line may be provided for assistance. The goal is for students to develop or recall the patterns when adding integers instead of relying solely on the number line.

Solutions:

Solutions will vary.

Adapted from Lawrence Burke

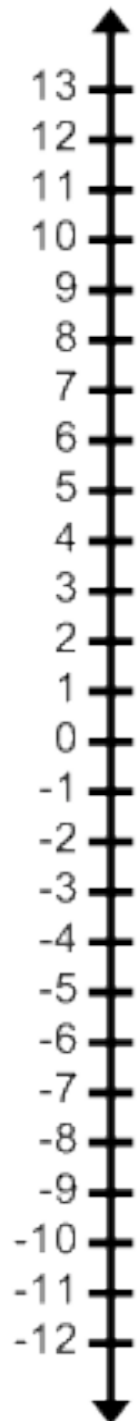
“Sign” your Name

Letter	Value
A	-12
B	-11
C	-10
D	-9
E	-8
F	-7
G	-6
H	-5
I	-4
J	-3
K	-2
L	-1
M	0

Letter	Value
N	+1
O	+2
P	+3
Q	+4
R	+5
S	+6
T	+7
U	+8
V	+9
W	+10
X	+11
Y	+12
Z	+13

Use the values for each letter in the charts above to find the amounts described below. Do not use a calculator. Use the provided number line and/or show your thinking.

1. The value of your first name:
2. The value of your middle name, if applicable:
3. The value of your last name:
4. The value of your entire name:
5. The absolute value of your first name:
6. The absolute value of your middle name, if applicable:
7. The absolute value of your last name:
8. The absolute value of your full name:
9. The value and absolute value of your teacher's last name:



Number Tricks

Common Core Standard:

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Additional Supporting Standard(s):

7.EE.1 Use properties of operations to generate equivalent expressions.

7.EE.4 Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Student Outcomes:

- I can connect order of operations to the decomposition of a multi-step equation.
- I can solve multi-step equations.
- I can translate words into mathematical expressions.
- I can translate mathematical expressions into words.

Materials:

- Phone Number Trick
- Number Trick Introduction PowerPoint
- Number Tricks Blackline from 2003 Resources for Algebra p. B-11 to use as model/overhead
- Algebra Tiles
- Number Tricks Handouts 1-4
- Four Column Solving Handout
- PDF files of the handouts can be downloaded at: <http://algebra.mrmeyer.com/> found in the Week 3 “handouts”. A PowerPoint has also been created to lead the lesson and can be found in the Week 3 “PowerPoint”.

Advance Preparation:

Algebra Tiles can be made available to students as manipulatives to model the Number Tricks as shown on the Number Tricks Blackline from 2003 resources for Algebra p. B-11

Directions:

1. Introduce the lesson by using the Phone Number Trick. Students will use a calculator to follow the directions and the end result should be their phone number. This is more of a “hook” than an instructive tool.
2. Use the Number Trick Introduction PowerPoint in conjunction with the Number Tricks Blackline from 2003 resources for Algebra p. B-11 and Algebra Tiles. Have students construct a model of the number trick. Extend this idea by allowing students to create and record their own Number Trick models.
3. Provide each student with the Number Tricks 1 Handout. A PowerPoint presentation that leads this task can be found at: <http://algebra.mrmeyer.com/> . The PowerPoint is not necessary but it is a nice visual for students to follow.
4. Each “Number Trick” on the handouts has its own unusual name. The first is “THE THREE SWAP”. Instruct students to enter a number between 1 and 25 in the top box of the TEST No. column. The corresponding box in the EQUATION column is x .

Example:

The Three Swap

TEST No.

EQUATION

5. Pick a number between 1 and 25

5

x

- Continue to lead the students through the instructions. See the answer key for the progression.
 - Step #3 has been left blank to keep students from working ahead. When your students are ready, give them the instruction for Step #3: Multiply the result by 3.
 - All students will end with the number 7 at the bottom of the TEST No. column if the steps were followed correctly.
6. The second number trick, “THE CORNER KICK” is completed the same way as “THE THREE SWAP”. You may decide to lead the students again or allow them to attempt to complete the trick without assistance. They will need to know Step #3 Multiply the result by 4. All students will end with the number they started with if done correctly.
 7. The third Number Trick has no name (students can give it the name of their choice). This Number Trick is different than the first two in that the students are given the equation progression and need to fill in the missing steps. All students will end with the number 0 if done correctly.
 8. “THE LEFTY NO-LOOKER” only gives students the end equation. Students should again choose an initial TEST No. and then complete the steps and equation progression without any other information. The goal here is that students use the strategy of working the order of operations backwards to decompose the equation. All students will end with the number of 27 if done correctly.
 9. IMPORTANT – the rest of the number tricks are a departure from the previous examples. Starting with “THE SHORT SLEEVES”, students DO NOT pick a number to start. Students are NOT to choose the initial TEST No., they are to *discover* the need to start with Step #5 and work backwards. All students will have an initial TEST No. of -39 if done correctly. The next 3 number tricks are completed in similar fashion.

Questions to Pose:Before:

- Have you ever followed instructions to put something together? How could you use those same instructions to take it apart?
- How is the order of operations similar to instructions for putting something together?
- How could the order of operations be used to take something apart?

During:

- What strategies did you use to complete the Number Trick?
- How is the equation column similar to the steps?
- How does the TEST No. progress?
- How can we start the LEFTY NO-LOOKER?
- How can you look at an expression and decide what operation to do last?
- How can you use the steps like instructions to put something together and then take it apart?

After:

- How does composing (building) an equation using the order of operations help you to decompose (take apart) that same equation.
- What does working an equation backward mean to you?

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
When completing a step with multiplication, students might not realize when parentheses are needed.	Model multiplying an expression with algebra tiles. Discuss the difference between “multiply the result by 3” and simply “multiply by 3”
Students may use the division symbol (\div) to notate a division step. This is acceptable but not desired.	Encourage students to represent division with a fraction bar and discuss what part of the expression needs to be above the fraction bar.

Special Notes:

The FOUR COLUMN SOLVING handout is an easier version of this task and is designed for additional practice. Students will likely need assistance on this handout when writing steps for the equation $5 - 7x = 40$.

- | | | |
|--|----|--|
| <ol style="list-style-type: none"> 1. Pick a number 2. Multiply by -7 3. Add 5 4. The result is 40 | OR | <ol style="list-style-type: none"> 1. Pick a number 2. Multiply by 7 3. Subtract the result from 5 4. The result is 40 |
|--|----|--|

Solutions: See Key

Taken from [Dan Meyer](http://algebra.mrmeyer.com/) - <http://algebra.mrmeyer.com/>

Phone Number Trick

Directions:

1. Key in the first three digits of your phone number, NOT the area code. If your number is 704-728-4567, the 1st 3 digits are 728.
2. Multiply by 80
3. Add 1
4. Multiply by 250
5. Add the last 4 digits of your phone number
6. Add the last 4 digits of your phone number again
7. Subtract 250
8. Divide number by 2
9. What is the result?

NUMBER TRICKS

Name: _____

Choose your own number.

THE THREE SWAP

TEST No.

EQUATION

1. Pick a number between 1 and 25.

2. Add 9 to it.

3. _____

4. Subtract 6.

5. Divide by 3.

6. Subtract your original number.

THE CORNER KICK

TEST No.

EQUATION

1. Pick a number between 1 and 25.

2. Add 7 to it.

3. _____

4. Subtract 16.

5. Divide by 4.

6. Subtract 3.

Taken from [Dan Meyer](http://algebra.mrmeyer.com/) - <http://algebra.mrmeyer.com/> (week 3 handouts)

NUMBER TRICKS

Name: _____

Choose your own number.

	TEST No.	EQUATION
1. Pick a number between 1 and 25.	<input type="text"/>	x
2. Multiply it by 9.	<input type="text"/>	$9x$
3. _____	<input type="text"/>	$9x - 18$
4. _____	<input type="text"/>	$\frac{9x - 18}{9}$
5. _____	<input type="text"/>	$\frac{9x - 18}{9} - x$
6. _____	<input type="text"/>	$\frac{9x - 18}{9} - x + 2$

THE LEFTY NO-LOOKER

	TEST No.	EQUATION
1. Pick a number between 1 and 25.	<input type="text"/>	x
2. _____	<input type="text"/>	
3. _____	<input type="text"/>	
4. _____	<input type="text"/>	
5. _____	<input type="text"/>	
6. _____	<input type="text"/>	$4 \left(\frac{x+7}{4} \right) + 5 - x$

Taken from [Dan Meyer](http://algebra.mrmeyer.com/) - <http://algebra.mrmeyer.com/> (week 3 handouts)

NUMBER TRICKS

Name: _____

Choose your own number.

THE SHORT SLEEVES

TEST No.

EQUATION

1. Pick a number.

2. _____

3. _____

4. _____

5. _____

6. _____

$$\frac{-2(x + 3) + 18}{6}$$

THE LEFTY NO-LOOKER

TEST No.

EQUATION

1. Pick a number.

2. _____

3. _____

4. _____

5. _____

6. _____

$$\frac{2(5x - 25) + 10}{5}$$

Taken from [Dan Meyer](http://algebra.mrmeyer.com/) - <http://algebra.mrmeyer.com/> (week 3 handouts)

NUMBER TRICKS

Name: _____

Choose your own number.

THE DUSTY HANDSHAKE

TEST No.

EQUATION

1. Pick a number.

2. _____

3. _____

4. _____

5. _____

6. _____

$$\frac{5(x+3)}{4} + 3$$

TEST No.

EQUATION

1. Pick a number.

2. _____

3. _____

4. _____

5. _____

6. _____

$$2\left(\frac{x+3}{7}\right) - 9$$

Taken from [Dan Meyer](http://algebra.mrmeyer.com/) - <http://algebra.mrmeyer.com/> (week 3 handouts)

FOUR COLUMN SOLVING

Name: _____

EQUATION	STEPS	ANSWER	CHECK
	<ol style="list-style-type: none">1. Pick a number.2. Multiply by 4.3. Subtract 2.4. The result is 10.		
$\frac{x-3}{7} = 6$			
	<ol style="list-style-type: none">1. Pick a number.2. Add 7.3. Multiply by 5.4. The result is -30.		
$5 - 7x = 40$			
		10	

Taken from [Dan Meyer](http://algebra.mrmeyer.com/) - <http://algebra.mrmeyer.com/> (week 3 handouts)

FOUR COLUMN SOLVING continued

EQUATION	STEPS	ANSWER	CHECK
	<ol style="list-style-type: none">1. Pick a number.2. Divide by 4.3. Subtract 8.4. The result is -1.		
$-5x - 2 = -12$			
	<ol style="list-style-type: none">1. Pick a number.2. Multiply by -2.3. Subtract 9.4. The result is -1.		
		-7	
$\frac{2x - 10}{5} = 8$			

Taken from [Dan Meyer](http://algebra.mrmeyer.com/) - <http://algebra.mrmeyer.com/> (week 3 handouts)

Sweet Algebra

Common Core Standard:

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

Student Outcomes:

- I can translate words into algebraic expressions.
- I can write and solve an equation to model a situation.
- I can use the arithmetic from a problem to generalize an algebraic solution.
- I can solve multi-step equations.

Materials:

- Sweet Algebra Handout
- 11 brown paper lunch bags.
- Individually wrapped candies (optional)

Advance Preparation:

- Students need prior exposure to solving multi-step equations.
- The brown paper lunch bags must be labeled. Write the variable X on the teacher bag. The rest of the bags will be numbered 1 through 10.
- Fill the bags with the given amounts of candies (if you choose to use actual candy):

Teacher's Bag: 6 Bag 1: 16 Bag 2: 18 Bag 3: 16 Bag 4: 19 Bag 5: 8

 Bag 6: 3 Bag 7: 19 Bag 8: 12 Bag 9: 20 Bag 10: 14

- Securely close the bags and display them in the room where students can see them. Do not let the students touch the bags.

Directions:

1. Each student will have his/her own Sweet Algebra Handout to complete but this task works best when students work as a group.
2. Introduce the task by explaining that the number of candies in the teachers bag is represented by the variable x . Students are given clues to determine the number of candies in each of the other 10 bags on the Sweet Algebra Handout.
3. Have students complete Part 1 of the handout.

4. Have students complete Part 2 of the handout using the expressions from Part 1 so that they can solve for x (the number of candies in the teacher's bag)
5. When students know the value of x , they should go back to Part 1 and use the expressions to determine the amount of candies in each bag.

Questions to Pose:

Before:

- What are similarities/differences between expressions and equations?
- What is the meaning of the equal sign?
- Where do you find expressions within equations?
- X is the variable representing the number of candies in the teacher's bag. Why might the term "variable" be misleading in this situation?

During:

- By looking at the expressions representing bag #6 and bag #2, can you determine which bag contains more candy? Explain.
- By looking at the expressions representing bag #4 and bag #7, can you determine which bag contains more candy? Explain why this is different than comparing bag #6 to bag #2
- What does solving for x mean for this task?
- How will you know you have solved an equation correctly in Part 2? How can solving two or more equations in Part 2 help you know you have solved an equation correctly?

After:

- How did knowing the value of x help you complete the task?
- What are similarities/differences between expressions and equations? How were they combined to help complete this task?

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Students may want to use variables in addition to x .	Lead students to write all expressions in relation to the number of candies in the teacher's bag (x)

Special Notes:

- If you decide to put candies in each of the 11 brown paper lunch bags, you will need a total of 151 candies. You can let groups pick the bag they want and share the candies once they have completed. Some teachers let the groups pick based on the order they finish. You may decide not to put candies in the bags and just use them as props.
- In Part 2, students are only required to write 3 of the possible 5 equations and then only required to solve 2 of those equations. You may want to require more based on the ability of your students.

Solutions: See Key

Partners for Mathematics Learning, 2009

Sweet Algebra



Part 1: How many candies are in each bag?

Candy Clues: Write an algebraic expression for each clue.

_____ The teacher has x number of candies in his/her bag.

_____ Bag #1 contains four more than twice the number of candies in the teacher's bag

_____ Bag #2 contains three times the number of candies in the teacher's bag

_____ Bag #3 contains 10 more candies than the teacher's bag

_____ Bag #4 has 5 less than 4 times the number of candies as the teacher

_____ Bag #5 has half as many candies as Bag #1

_____ Bag #6 has $\frac{1}{6}$ the number of candies in Bag #2

_____ Bag #7 has one more than three times the number of candies in the teacher's bag

_____ Bag #8 contains four less candies than Bag #3

_____ Bag #9 has twice the sum of the amount of candies in the teacher's bag and four

_____ Bag #10 has six more than 50% of the candies in bag #3

Partners for Mathematics Learning, 2009

Part 2: How many candies are in the teacher's bag?

Edible Equations: Write at least three algebraic equations that you can use to find the number of candies in the teacher's bag using the clues below.

The total number of candies in Bags #3, #5, and #9 is 44.

The total number of candies in Bags #2, #4, and #8 is 49.

The difference in the amount of candy in Bag #7 and Bag #1 is 3.

Bag #7 and Bag #4 have the same number of candies.

Bag #9 has one more piece of candy than Bag #4.

Solve two equations to find the number of candies in the teacher's bag.

Which bag would you want? Explain why.

Partners for Mathematics Learning, 2009

Sweet Algebra



Part 1: How many candies are in each bag?

Candy Clues: Write an algebraic expression for each clue.

 x The teacher has x number of candies in his/her bag.

 $2x + 4$ Bag #1 contains four more than twice the number of candies in the teacher's bag

 $3x$ Bag #2 contains three times the number of candies in the teacher's bag

 $x + 10$ Bag #3 contains 10 more candies than the teacher's bag

 $4x - 5$ Bag #4 has 5 less than 4 times the number of candies as the teacher

 $\frac{2x + 4}{2} = x + 2$ Bag #5 has half as many candies as Bag #1

 $\frac{1}{6}(3x) = \frac{1}{2}x$ Bag #6 has $\frac{1}{6}$ the number of candies in Bag #2

 $3x + 1$ Bag #7 has one more than three times the number of candies in the teacher's bag

 $x + 6$ Bag #8 contains four less candies than Bag #3

 $2(x + 4) = 2x + 8$ Bag #9 has twice the sum of the amount of candies in the teacher's bag and four

 $.5x + 6$ Bag #10 has six more than 50% of the candies in bag #3

Partners for Mathematics Learning, 2009

Part 2: How many candies are in the teacher's bag?

Edible Equations: Write at least three algebraic equations that you can use to find the number of candies in the teacher's bag using the clues below.

The total number of candies in Bags #3, #5, and #9 is 44.

$$\begin{aligned}x + 10 + x + 2 + 2(x + 4) &= 44 \\4x + 20 &= 44 \\4x &= 24 \\x &= 6\end{aligned}$$

The total number of candies in Bags #2, #4, and #8 is 49.

$$\begin{aligned}3x + 4x - 5 + 2x &= 49 \\9x - 5 &= 49 \\9x &= 54 \\x &= 6\end{aligned}$$

The difference in the amount of candy in Bag #7 and Bag #1 is 3.

$$\begin{aligned}3x + 1 - (2x + 4) &= 3 \\3x + 1 - 2x - 4 &= 3 \\x - 3 &= 3 \\x &= 6\end{aligned}$$

Bag #7 and Bag #4 have the same number of candies.

$$\begin{aligned}3x + 1 &= 4x - 5 \\6 &= x\end{aligned}$$

Bag #9 has one more piece of candy than Bag #4.

$$\begin{aligned}2(x + 4) &= 4x - 5 + 1 \\2x + 8 &= 4x - 4 \\12 &= 2x \\6 &= x\end{aligned}$$

Solve two equations to find the number of candies in the teacher's bag.

Which bag would you want? Explain why.

Bag #9 because it has the most candies. Each equation above shows that the teacher's bag has 6 candies. Bag #9 has twice the sum of the amount of candies in the teacher's bag and four. Expressed algebraically: $2(6+4) = 20$

Partners for Mathematics Learning, 2009

Murphy to Manteo

Common Core Standard:

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Additional/Supporting Standard(s):

7.RP.2 Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

- I can compute actual lengths from a scale drawing
- I can compute fractional parts and use them for real world problems

Materials:

- North Carolina road maps for each group (2-3 students in a group)
- Ruler
- Handout “Murphy to Manteo”

Advance Preparation:

- Obtain North Carolina road maps – one for each group of 2-3
- Be sure students are proficient in locating and using the scale on a map
- Discuss with students appropriate measures to use with the map

Directions:

1. Put students in groups of two or three for this task.
2. Hand out the maps and give students time to study the map and locate the scale
3. Have students work in groups to complete the activity “Murphy to Manteo”
4. Allow time at the end of the lesson for students to share their findings and the method they used to arrive at their answers

Questions to Pose:

Before:

- What is the difference in traveling “as the cardinal flies” and on the roads?
- What does the scale on a map tell us?
- What is the scale on the map you are using?

During:

- How accurately do you need to measure your distances?
- How did you decide what unit of measure to use?
- How can you calculate the fraction of the roads that are interstate?
- Is the distance the cardinal flies different from the distance by highway? Why?
- How do you use the speed limit to help you estimate the time it will take you to travel?

After:

- What is the ratio of the distance on the map:actual distance?
- How did you use this ratio to answer the questions?
- Was this the same each time you measured?
- How does the scale on the map compare to unit rate? (unit rate covered in sixth grade)
- How did you use the speed limit to determine the time it would take you to travel?

Possible Misconceptions/Suggestions:

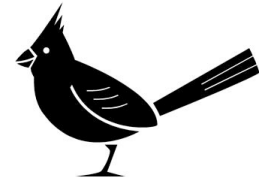
Misconception	Suggestion
Students do not measure in units appropriate for the scale on the map	Have a discussion with the class prior to the activity about reading the scale and determining the measurement. The scale on the map will determine the units that the student will measure in.

Special Notes:

- You may need to adjust the questions depending on your location. For example if you are in Wake County, you may want to change the location for question 1.
- Students can use string to measure the distances by the road and then measure the string.

Solutions: NA

Murphy to Manteo



Answer the questions below “as the cardinal flies”, that is, measure straight line distances between the points. Use the scale on the map to find the distances. For each question, show how you arrived at your answer.

1. How far is it from Raleigh to your hometown?
2. You are a great lover of the Hatteras Lighthouse. You are in Wilmington and decide to make the trip to see the lighthouse. How far will you travel?
3. The mountains are more your taste so explore the high country. How far will you travel from Chimney Rock Park near Lake Lure to the highest peak east of the Mississippi, Mount Mitchell?
4. You are a big basketball fan. How far will it be from your hometown to Chapel Hill to see the Tarheels?
5. Oops, you are a Blue Devil fan!! How far from your hometown to Durham to catch the Blue Devils?
6. You are a movie fan. How far from Asheville where *The Hunger Games* were filmed to Mount Airy, the town that inspired *The Andy Griffith Show*?
7. Lexington is famous for their barbeque while Morehead City is famous for its seafood. How far would you travel to have barbeque for lunch and seafood for dinner?
8. Pepsi Cola was invented in New Bern. Salisbury is the home of Cheerwine. If you start in New Bern and see the Pepsi Museum and travel on to Salisbury to the Cheerwine Museum, how far will you travel?
9. You like science? How far from Discovery Place in Charlotte to the Wright Brother’s museum in Kitty Hawk?
10. Now stretch from Murphy to Manteo. How far across the state on the cardinal’s back?
11. Cardinals can fly up to 50mph. Assuming that he flies at his max speed how long will it take you to get to each destination above?

NOW.....

- Pick a place that you would like to visit in North Carolina. How far would the trip be if you traveled on the bird’s back? How long will the trip take you?
- If you traveled on the roads, how much farther would this trip be? Calculate the fraction of your trip that is on interstate roads and the fraction that is on state roads. How many miles will you travel on each road?

Slicing Pi

Common Core Standard:

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.2 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

- I can describe the relationship between radius and diameter of a circle
- I can find the area of a circle
- I can use the area of a circle to approximate the radius or diameter
- I can relate the area of a circle to its circumference



Materials:

- Scissors
- Glue
- Handout – “slice of pi”
- Handout – Slice of Pi Questionnaire

Advance Preparation:

- Review with the class the meaning of area and how to calculate the area of a rectangle and a circle
- Make a copy of the handouts for each student
- Make an example of the cut apart circle to show students

Directions:

1.  Students should cut apart the slices of the circle and rearrange them to make a parallelogram as shown in the diagram.
 2.  Cut half the end piece off and put on the other end to create a rectangle as shown below
 3. Glue the pieces to a sheet of paper to make a rectangle.
 4. Students should work with a partner to complete the Slice of Pi questionnaire.
 5. Once students have had time to work together, discuss their answers as a class.
-

Questions to Pose:Before:

- What is the area of the circle in terms of triangles?
- What is the radius of the circle?
- How can we use the triangles to create a parallelogram?

During:

- Did we change the area of the circle when we took it apart and made a parallelogram?
- Did we change the area when we altered the parallelogram to make a rectangle?
- Can we give an exact area of the circle or the rectangle?
- Why do we give the area in terms of Pi?
- How can you prove that the area of the circle and the area of the rectangle are the same?

After:

- What is the relationship between the radius and the diameter of a circle?
- What is the relationship between the circumference of a circle and its area?
- If you know the area of the circle, how can you find the diameter or the radius?

Possible Misconceptions/Suggestions:

Misconceptions	Possible solutions
Students believe that you can get an exact number for the area if you use 3.14 for Pi	Use a calculator with a Pi key to calculate the area of the circle
Students see the circle as larger than the rectangle and not related in size	Cutting apart the circle and rearranging it into a rectangle helps alleviate this Repeat the process with different sized circles

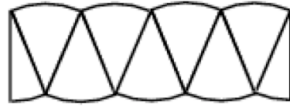
Special Notes:

The activity gives students a visual representation of the area of the circle rather than memorizing a formula. The activity can be extended by using circles of different sizes (the master circle can be reduced and enlarged with a copier to retain the triangles).

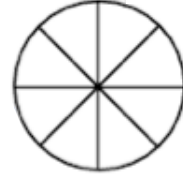
Solutions:

See attached Blackline master

A Slice of Pi Questionnaire



1. What part of the circle now represents the width of the rectangle?



2. How can you describe the length of the rectangle in terms of the circle?

3. How do you find the circumference of a circle?

4. In terms of Pi, how can you describe the length of the rectangle?

5. How do you find the area of a rectangle?

6. In terms of Pi, what is the formula for the area of the rectangle?

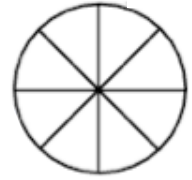
7. In terms of Pi, what is the area of your rectangle?

8. In terms of Pi, what is the area of your circle?

9. If instead of 3 units, you use the variable r as the radius, what would the area of the rectangle be? What would the area of the circle be?

10. Based on what you know about area, explain why you got these results.

A Slice of Pi Questionnaire



1. What part of the circle now represents the width of the rectangle?

The radius (r)

2. How can you describe the length of the rectangle in terms of the circle?

$\frac{1}{2}$ Circumference of the circle

3. How do you find the circumference of a circle?

$2 \cdot \pi \cdot r$

4. In terms of Pi, how can you describe the length of the rectangle?

Half the circumference or $\frac{1}{2}(2 \cdot \pi \cdot r)$ or πr

5. How do you find the area of a rectangle?

Length \cdot Width

6. In terms of Pi, what is the formula for the area of the rectangle?

Length (πr) \cdot Width (r) or πr^2

7. In terms of Pi, what is the area of your rectangle?

9π

8. In terms of Pi, what is the area of your circle?

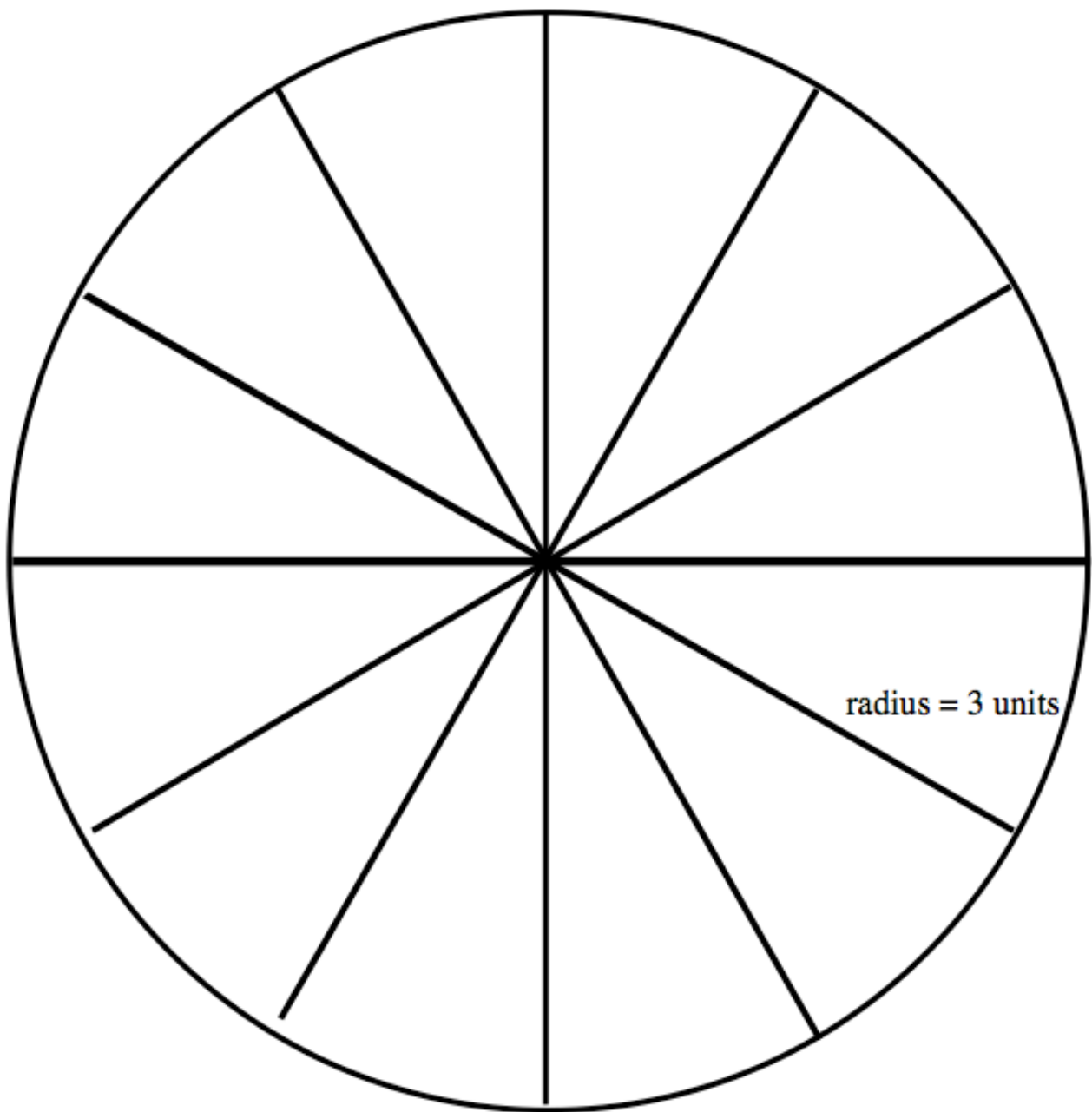
9π

9. If instead of 3 units, you use the variable r as the radius, what would the area of the rectangle be? What would the area of the circle be?

πr^2

10. Based on what you know about area, explain why you got these results.

Students should refer to the fact that the area is the space inside and although they altered the shape, they did not add to or take away from the area of either



Changing Surface Areas

Common Core Standard:

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Additional/Supporting Standard(s):

7.RP.1, 7.RP.2 Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

- I can solve real world problems involving surface area of cubes
- I can draw a model for a proportional relationship
- I can compute unit rate

Materials:

- 125 centimeter cubes for each group of students
- ***Changing Surface Areas*** handouts
- Centimeter grid paper (if students need to make nets)

Advance Preparation:

- Review with students the meaning of area and unit rate (covered in sixth grade)
- Put cubes for each group in a bag making sure there are exactly 125 cubes
- Copy the handout for each group

Directions:

1. Put students in groups according to the number of cubes you have available. Groups of two work best.
2. As a whole group activity, do parts 1 and 2 of the handout. Be sure that the students are proficient with the concept of the unit cube.
3. Direct the groups to continue to build the cubes and fill in the chart. Once they run out of cubes, the group will have to discuss and decide how to fill in the chart.
4. Leave time at the end of the lesson for discussion of the “after” questions

Questions to Pose:

Before:

- How do we calculate the area of a square?
- What are the faces of a three dimensional figure?
- What is meant by the term unit rate?
- How would you describe the surface area of a figure?

During:

- Does the area of the face of the unit cube ever change?
- Does the area of the face of the cube you build differ from the area of the face of the unit cube?
- How is the area of the face related to the edge of the cube?
- Does the number of faces change as you are building the cubes?
- Do all cubes have the same number of faces?
- Are you seeing a pattern in the simplified ratios?
- Could you use the patterns to predict the simplified ratio of any size cube?

After:

- Predict the simplified ratio of a cube that has an edge of 12 cm? 20 cm? 30cm?
- How can you use this ratio to predict the surface area of a cube?
- If the cubes were 2 cm cubes instead of 1 cm, would you get the same results? Why or why not?
- How do the cubes relate to the nets that you did in sixth grade?
- How can you relate the surface area that you calculated to the surface area you would count in a net?
- What measure does the total number of cubes that you used to build the figure model?
- What are some examples of cubes in real life?
- Using what we have discovered, how could we determine the surface area of these cubes?

Possible Misconceptions/Suggestions:

Possible Misconception	Suggestions
Students fail to count the face on the bottom of the cube	Before students begin the task, visually show a cube and turn it so all the faces have been seen and counted
Students confuse the surface area and the volume of the cube	Before the students begin the task, have a class discussion about the surface area and volume, using a cube to demonstrate

Special Notes:

- If you do not have access to centimeter cubes, snap cubes can be used and the measurements can be in units rather than centimeters
- Be sure to allow students ample time to build as many cubes as possible with the 125 cubes
- Time for discussion of the questions is very important in this task
- The task can be extended to include the volume of the cube as well

Solutions: See attached BLM



Changing Surface Area



Use one cube. Call it the unit cube.

1. How many faces are on the cube?
2. A 1cm by 1cm square has an area of 1 cm^2 .
3. The unit cube has a total surface area of _____.

Build a model of a cube with each edge 2 cm. You should have used 8 cubes.

1. How many faces are on the cube? _____
2. What is the area of each face? _____
3. What is the surface area of this cube?

$$\underline{\hspace{2cm}} \# \text{ of faces} \cdot \underline{\hspace{2cm}} \text{ area of face} = \underline{\hspace{2cm}}$$

Continue to build larger cubes with edges of 3cm then 4 cm then 5 cm...until you run out of cubes. Find the surface area of each cube you build and record it in the chart. See if you can finish the chart up to a cube that has an edge of 10cm.

Changing Surface Area

Edge of cube (cm)	Total cubes used	Area of one face (cm ²)	Surface area of cube (cm ³)	Ratio of surface area of large cube to unit cube	Simplified ratio (unit rate)
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Changing Surface Area

Edge of cube (cm)	Total cubes used	Area of one face (cm ²)	Surface area of cube (cm ³)	Ratio of surface area of large cube to unit cube	Simplified ratio (unit rate)
1	1	1	6	6 : 6	1 : 1
2	8	4	24	24 : 6	4 : 1
3	27	9	54	54 : 6	9 : 1
4	64	16	96	96 : 6	16 : 1
5	125	25	150	150 : 6	25 : 1
6	216	36	216	216 : 6	36 : 1
7	343	49	294	294 : 6	49 : 1
8	512	64	384	384 : 6	64 : 1
9	729	81	486	486 : 6	81 : 1
10	1000	100	600	600 : 6	100 : 1

Packing to Perfection

Common Core Standard:

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Student Outcomes:

- I can find the surface area of a rectangular prism.
- I can connect the model of a rectangular prism to the formula for its volume using the area of the base and the height.
- I can recognize that orientation does not affect the volume.
- I can recognize that prisms can have the same volume but different dimensions and surface areas.

Materials:

- Packing to Perfection Handout
- Snap Cubes (24 per group of students)

Advance Preparation:

- Students must know the concept of volume and surface area but the formulas are not required.
- Snap Cubes (or any type of cube) must be in groups of 24.

Directions:

1. Give each pair or group of students, 24 snap cubes (linking cubes, pop cubes, etc.).
2. Provide each student with the handout, Packing to Perfection.
3. Read the scenario to students and answer any questions that may be asked.
4. Allow students ample time to create their packages and calculate the surface areas. (You may or may not allow students to use calculators. You might want students to physically count surface area.)
5. Have groups share their findings with the class.

Questions to Pose:

Before:

- Is there a relationship between surface area and volume?
- Can rectangular prisms with different dimensions have the same volume?
- Do rectangular prisms with the same volume also have the same surface area?

During:

- What do you notice about the relationship between the dimensions of the rectangular prisms and their volumes? (*factors of 24*)
- What is the area of the base of one of your packages? How many truffles will it take to fill one layer of your package? How many layers will it take to fill your package?
- If you have 12 truffles in the bottom of your package, how many layers of candy does the package hold?
- If you have 4 truffles in the bottom of your package, how many layers of candy does the package hold?
- If the package is 8 layers high, how many truffles in each layer?

After:

- Which of your packages requires the least amount of material?
- How could a company save money when designing the packaging of their product?
- Why is the surface area smaller as your rectangular prism starts to look more like a cube?

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Surface area is the same for all rectangular prisms when the volume is equal.	Have students find surface area by counting the number of faces on each figure they create.

Special Notes:

- “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of *why* the formula works and how the formula relates to the measure (area and volume) and the figure.
- Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume.
- The surface area formula is not the expectation with this standard. Students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area.

Solutions: Answer key is provided at end of task.

Packing to Perfection

Is there a relationship between surface area and volume? Can rectangular prisms with different dimensions have the same volume? Do rectangular prisms with the same volume have the same surface area?

Take 24 snap cubes and imagine that each cube is a fancy chocolate truffle. For shipping purposes, these truffles need to be packaged in boxes that are rectangular prisms. Knowing the company only sells their truffles in groups of 24, what are the possible dimensions for the boxes?

Record your information in the table below.

Length	Width	Height	Volume	Surface Area
			24 cubic units	
			24 cubic units	
			24 cubic units	
			24 cubic units	
			24 cubic units	
			24 cubic units	

1. Which of your packages requires the least amount of material? The greatest amount of material? Why is the amount of material important?

2. What do you notice about the shape of the package with the smallest surface area? How about the package with the greatest surface area?

3. Which package would you recommend to the chocolate company? Why?

4. If sold to a store, how would you suggest that the package be displayed on the shelf? Why?

5. What is the relationship between surface area and volume?

6. Can rectangular prisms with different dimensions have the same volume?

7. Do rectangular prisms with the same volume have the same surface area?

Packing to Perfection

Is there a relationship between surface area and volume? Can rectangular prisms with different dimensions have the same volume? Do rectangular prisms with the same volume have the same surface area?

Take 24 snap cubes and imagine that each cube is a fancy chocolate truffle. For shipping purposes, these truffles need to be packaged in boxes that are rectangular prisms. Knowing the company only sells their truffles in groups of 24, what are the possible dimensions for the boxes?

Record your information in the table below.

Length	Width	Height	Volume	Surface Area
1	1	24	24 cubic units	98 units ²
2	1	12	24 cubic units	76 units ²
3	1	8	24 cubic units	70 units ²
4	1	6	24 cubic units	68 units ²
2	2	6	24 cubic units	56 units ²
2	3	4	24 cubic units	52 units ²

1. Which of your packages requires the least amount of material? The greatest amount of material? Why is the amount of material important?
 - The 2x3x4 package requires the least amount of material
 - The 1x1x24 package requires the greatest amount of material
 - The amount of material is important because material costs money and more material = high cost.
2. What do you notice about the shape of the package with the smallest surface area? How about the package with the greatest surface area?

Possible answers:

 - The shape of the package with the smallest surface area is the package that most closely resembles a cube.
 - The package with the greatest surface area is the package that is least like a cube.

3. Which package would you recommend to the chocolate company? Why?

Possible answer:

- I would recommend the 2x3x4 package because it requires the least amount of material to cover the package. The company could save money by using the least amount of material possible.

4. If sold to a store, how would you suggest that the package be displayed on the shelf? Why?

Answers will vary

5. What is the relationship between surface area and volume?

- Surface area decreases as the rectangular prisms move closer to the shape of a cube.

6. Can rectangular prisms with different dimensions have the same volume?

- Yes, we found six different rectangular prisms with a volume of 24 units³. All prism had a length, width, and height that multiplied together to make 24.

7. Why did the rectangular prisms with the same volume have a different surface area?

- The number of exposed faces of each unit cube is different for each prism. For example, the 1x1x24 prism has 22 cubes with four exposed faces and the two end cubes have five exposed faces. The 2x3x4 has 8 cubes with three exposed faces (the corners), 12 cubes with two exposed faces, and 4 cubes with one exposed face.

X Marks the Spot

Common Core Standard:

Draw informal comparative inferences about two populations.

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Additional/Supporting Standards: 7.SP.4

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

Student Outcomes:

- I can create a dot plot.
- I can find mean, median, and mode.
- I can find the lower and upper extremes, lower and upper quartiles, and the range.
- I can create a box plot.
- I can interpret the data in a box plot.

Materials:

- X Marks the Spot handout
- Grid paper (cm)
- Sticky notes
- Two blank number lines on the board or two large pieces of paper posted on the wall for a class dot plot.

Advance Preparation:

- Students should be familiar with the parts of a box plot and be able to create one from a set of data. If students are not familiar with box plots, you can use the Instructional Guide found on the last page of this document as an introduction before beginning this task.

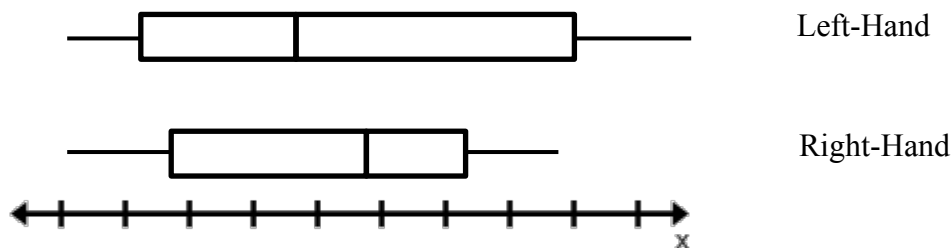
Directions:

1. Give each student the X Marks the Spot handout and two sheets of blank cm grid paper.
2. Establish rules about how all students are expected to make the X's. For example, each line of the X must completely cross the cm square on the grid paper before making the next X.



3. Have students predict the number of X's they will be able to write in 30 seconds. Record that information on the handout.
4. Have students mark an X in each box using their right hand for 30 seconds. The teacher will dictate start and stop time.
5. Students should record their number of X's on a sticky note. Have students use a dark marker and write as large as possible when recording their number because students will need to read this from their seats later in the task.
6. Have students create a "dot plot" on the board or wall with the sticky notes. A discussion about the number line intervals may need to occur before creating the number line that will contain the data. Typical results should range from 20-60 X's.
7. Find the mean, median, and mode of the class data.
8. Find the lower and upper extremes, lower and upper quartiles, and the interquartile range.
9. Create a box plot of the data on the board.
10. Repeat the activity, but now have students use their left hand to mark X's in each box on the cm grid paper.
11. Record the number of X's on a sticky note. Repeat the data collection from above.
12. Create a second box plot above or below the first plot. (Stacked box plot)

Example:



Questions to Pose:

Before:

- Make a prediction: Will you have more X's marked with your right hand or left hand?
- Make a prediction: Will the mean be higher for right-handed or left-handed?

During:

- Compare the two box plots. Describe what you see.
- Which set of data has the greater variability? Why is this so?
- Which set of data has the smaller interquartile range? Why is this so?

After:

- From the two sets of data, can you make a conclusion about the number of right-handed students versus the number of left-handed students in this class?
- What are the strengths and limitations of dot plots when used to analyze data?
- What are the strengths and limitation of box plots when used to analyze data?
- What are the strengths and limitations of measures of center? Variability?
- Why would stacked box plots be beneficial when comparing two large sets of data? (not necessarily large numbers but a lot of numbers)

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Quartiles with larger ranges contain more data within the quartile.	Use the example at the beginning of the instructional guide found at the end of this task. This will provide a visual that quartiles contain the same amount of data but the range can be different.

Special Notes:

- Possible Extension: Calculate the percentage of right-handed and left-handed students in the class. Compare this with the conclusions students made from the collected data.

Solutions:

Will vary based on individual class data.

Adapted from *Partners for Mathematics Learning, 2009*

X Marks the Spot

PART 1

1. Prediction for the number of X's with right hand _____

Using your right hand, mark X's in the boxes on your cm grid paper. The teacher will tell you when to start and stop.

Actual number of X's marked with the right hand _____

2. Using the class data, complete the table below.

RIGHT-HANDED DATA	
Mean	
Median (Q2)	
Mode	
Lower Extreme	
Upper Extreme	
Range	
Lower Quartile (Q1)	
Upper Quartile (Q3)	
Interquartile Range	

3. Prediction for the number of X's with left hand _____

Using your left hand, mark X's in the boxes on your cm grid paper. The teacher will tell you when to start and stop.

Actual number of X's marked with left hand _____

4. Using the class data, complete the table below.

LEFT-HANDED DATA	
Mean	
Median (Q2)	
Mode	
Lower Extreme	
Upper Extreme	
Range	
Lower Quartile (Q1)	
Upper Quartile (Q3)	
Interquartile Range	

PART 2

Using the two sets of data, create a stacked box plot on graph paper. Draw one box plot to display the right-hand data and stack the other box plot to display the left-hand data.

Analyze your data by completing the following:

1. Which set of data has the greater median? How can you tell this by looking at your stacked box plot?
2. Which set of data has the greater mean? Explain why you think this mean is accurate.
3. Which set of data has the greatest range? Do outliers affect the range? Who are the outliers?
4. Analyze the data by describing the shape (clusters, peaks, symmetry) and variability (maximums, minimums, and outliers).
5. Interpret the data and be ready to discuss your findings.

Instructional Guide

Teacher Note: Only use if students are not familiar with box plots or need extra practice before completing X Marks the Spot.

1. Select 11 students to stand up and arrange themselves in order of height, from shortest to tallest.
2. The middle person represents the **median**, or **Q2**.
3. The middle person in the line of people *shorter* than the median represents the **lower quartile**, or **Q1**.
4. The middle person in the line of people *taller* than the median represents the **upper quartile**, or **Q3**.
5. The shortest and tallest students are the **lower and upper extremes**, respectively.
6. Repeat the process with 10 students to not differences in how the quartiles are found.

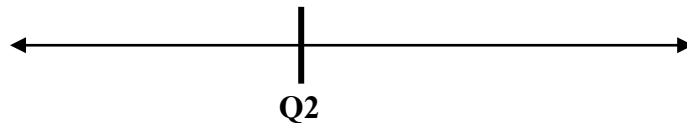
Creating a Box Plot

Use data to find interquartile range and create a box plot. Here are the steps to follow:

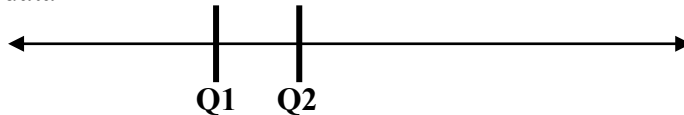
Before constructing a box plot

- Order the data from least to greatest and find the **median (Q2)**.
- Next find the **lower quartile (Q1)**. It is the median of the set of numbers *below* the median of the entire set of data.
- The **upper quartile (Q3)** is the median of the set of numbers *above* the median of the entire set of data.

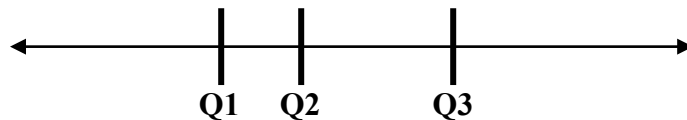
Step 1: Order the data from least to greatest and find the **median (Q2)**. Draw a number line, indicating the least and greatest values, and indicate the median (indicate the value of Q2)



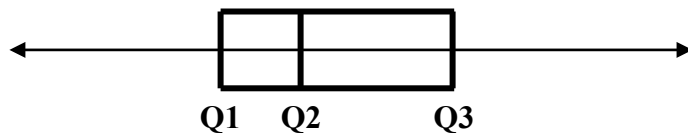
Step 2: Find the **lower quartile (Q1)**. It is the median of the set of numbers *below* the median of the entire set of data



Step 3: The **upper quartile (Q3)** is the median of the set of numbers *above* the median of the entire set of data.



Step 4: Draw a rectangle connecting Q1 and Q3 to show the interquartile.



The interquartile range is the difference between Q3 and Q1. The interquartile is the middle 50% of the data.

Step 5: Place dots to represent the **Lower Extreme (smallest value)** and the **Upper Extreme (largest value)**. Draw a line from the lower extreme to Q1 and a line from the upper extreme to Q3.

