TRANSFORMING NAD 27 AND NAD 83 POSITIONS: MAKING LEGACY MAPPING AND SURVEYS GPS COMPATIBLE

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Thomas H. Meyer
Robert Baron

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16. Abstract

The Connecticut Department of Transportation (CTDOT) and the University of Connecticut are creating a real-time network (RTN) to make real-time surveying widely available in Connecticut. This RTN uses global navigation satellite system (GNSS) technology (such as the U.S. Global Positioning System [GPS]) and is inherently most compatible with the North American Datum of 1983 (NAD83). An RTN will be most effective if it can be used throughout the State. There are still towns in Connecticut that have maps and surveys based on the North American Datum of 1927 (NAD 27), and global navigation satellite system (GNSS) surveying instruments cannot provide real-time positioning in NAD 27 with the accuracy levels demanded by land surveying for lack of a high-accuracy reference-frame transformation between NAD 27 and the reference frame of the GNSS (one of the modern International Terrestrial Reference Frames). Re-mapping and re-surveying the legacy maps is cost prohibitive. Therefore, the least-squares method was used to try to provide a transformation. We used weighted and non-weighted, ordinary and total least squares on both geocentric Cartesian (Earth-centered, Earth-fixed) positions and on positions in the Connecticut State Plane coordinate system. No transformation was best so, by parsimony, we recommend the model producing using ordinary least squares in State Plane Coordinates (SPC). The transformed coordinates' residuals differed from control values in eastings by 0.040 m (0.159 m 1-sigma) and by 0.001 m (0.128 m 1-sigma) in northings. Although inadequate for many surveying and engineering projects, this result should be adequate for topographic mapping and resource-location projects throughout Connecticut.

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SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
LENGTH				
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
AREA				
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
VOLUME				
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
NOTE: volumes greater than 1000 L shall be shown in m^3				
MASS				
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
TEMPERATURE (exact degrees)				
°F	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	°C
ILLUMINATION				
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
FORCE and PRESSURE or STRESS				
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
LENGTH				
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
AREA				
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
VOLUME				
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
MASS				
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
TEMPERATURE (exact degrees)				
°C	Celsius	1.8C+32	Fahrenheit	°F
ILLUMINATION				
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
FORCE and PRESSURE or STRESS				
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

*SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380.
(Revised March 2003)

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Symbols

$\boldsymbol{P}_{83}$	position in NAD 83, either SPC or ECEF (m)
$\boldsymbol{R}$	rotation matrix
$\boldsymbol{P}_{27}$	position in NAD 27, either SPC or ECEF (m)
$\boldsymbol{T}$	translation vector
$\boldsymbol{B}$	vector of observations (coordinates)
$\hat{\boldsymbol{\beta}}$	vector estimated parameters
$\boldsymbol{A}$	least-squares design matrix
$\boldsymbol{W}$	least-squares weight matrix

Introduction

There are still Connecticut towns (Connecticut is comprised of 169 autonomous towns [not townships]) that have maps and surveys based on the North American Datum of 1927 (NAD 27). A U.S. Global Positioning System (GPS) real-time surveying instrument produces positions in the coordinate system of the NAVSTAR orbits, which is currently the GPS-week 1674 realization of the World Geodetic System 1984 (WGS 84 (G1674)) (Wong et al. 2012). Currently, if NAD 27 positions are needed from a GPS, a Connecticut surveyor must either localize or perform a static survey controlled with NAD 27 markers. NAD 27 coordinates vary significantly in quality, and errors are automatically introduced if differing control points are used rather than the original project control. This presents a very significant problem to the surveyor who is trying to "follow in the footsteps" of a previous surveyor.

There are three general options for establishing compatibility between datums (Bauer and Burkholder 1996): (1) resurvey existing survey control points using modern techniques and technology, (2) abstract[1] historic observations and re-compute the coordinates for each point based on the datum-specific published coordinates of the fixed control points, and (3) model the differences between the datums to establish a reliable transformation. The modeling approach is the most practical option; the others are relatively expensive in time and money. The Connecticut Department of Transportation (CTDOT) and the University of Connecticut are creating a real-time network (RTN) to make real-time GNSS surveying widely available in Connecticut. This RTN will be most effective if it can be used throughout the State, so a transformation between WGS 84 (G1674) – the reference frame of the GPS – and NAD 27 is desired. (In this report the term **reference frame** is used synonymously with **datum**.) However, there are no control markers in Connecticut with WGS 84 (G1674) coordinates so computing such a transformation is infeasible.

A RTN produces coordinates in WGS 84 (G1674) because a GPS receiver uses the positions of the NAVSTAR GPS satellites as control, and the GPS satellites' positions are given in the broadcast ephemerides in WGS 84 (G1674). Wong et al. (2012) provided a high-accuracy transformation from WGS 84 (G1674) to the International GNSS Service 2008 Realization (IGS08) (Rebischung et al. 2012), and the latest realization of the North American Datum of 1983 (NAD 83 (2011)) is, in fact, just a 14-parameter Helmert transformation of IGS08. Therefore, a GPS receiver can (and does) already produce NAD 83 (2011) coordinates by transforming the WGS 84 coordinates. The transformation between NAD 27 and NAD 83 is the missing link.

There are many realizations of NAD 83. The first was NAD 83(1986), and it was the first geocentric reference frame in the world. It was able to be geocentric because it was constructed using space geodesy methods. When using space geodesy, ground points are positioned using trilateration from orbiting artificial satellites to the ground receiver. The satellites orbits around the geocenter so the geocenter is the logical origin for a space-geodetic reference frame. Furthermore, geocentric reference frames are inherently three-dimensional. They are Cartesian with their *Z*-axis defined as Earth's conventional rotational axis, and the *X*- and *Y*-axes define the equatorial plane. These *XYZ* coordinates (also known as Earth-Centered, Earth-Fixed [ECEF]) are converted to geodetic longitude, latitude, and ellipsoid height using formulas. In contrast, the

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[1] Here the word "abstract" is a verb that describes the process of combing the measurement records and summarizing the observations to the point they can be used to recompute a network based upon "different" or modern control point values. (E. Burkholder, pers. comm. 2013)

origin for NAD 27 is a survey marker in Kansas, and NAD 27 positions have no heights of any kind, so NAD 27 is a strictly horizontal datum. Since the first realization, NGS has frequently updated NAD 83 to keep pace with the rapid improvements in GPS positioning and space-geodesy methods. Each realization is more realistic than the last, meaning that physical measurements of distance and direction between markers matches ever more closely with those quantities calculated from coordinates (inversing). The latest NAD 83 realization was released in 2012, and it is called NAD 83(2011) because it was calculated from observations collected no later than 2011.

The NAD 83 realizations removed much of the distortion that exists in NAD 27 (Vogel 1986, Shreshtha 1990), which is why positions in the NAD 83 realizations, the WGS 84 realizations, and the realizations of the International Terrestrial Reference System (ITRS) can be transformed from any frame to another using a simple mathematical formula, the Helmert transformation. Based on Connecticut's small geographic size, if NAD 27 coordinates in Connecticut are sufficiently accurate either as a whole or, possibly, when divided in to zones, then a Helmert transformation can be found between NAD 27 and NAD 83 at surveying accuracy. Helmert transformations between NAD 83 and NAD 27 were given in Vancek and Steeves (1996), Vancek et al. (2002), and Wade and Doyle (1987). However, these transformations do not produce survey-accuracy results in general but they pertain to the whole of both datums, which is a much larger area than is needed here.

A Helmert transformation is very simple, just a single equation that is applied everywhere. To be successful, a Helmert transformation depends on whatever discrepancies there might be between the coordinate systems to be quite uniform, that the difference can be captured everywhere with a scale change, a translation of origin, and with a rotation. If a single transformation is inadequate, the regions can be divided into zones that get their own transformation. The limiting case is to have a very large number of zones, such as one zone for only a few markers. Such transformations are possible using polynomial surfaces, like splines. Shrestha (1987) and Shrestha and Dewitt (1989) discussed and evaluated several Helmert transformation methods and surface-fitting methods. The National Geodetic Survey (NGS) makes available NADCON, a computer program to transform between NAD 27 and NAD 83 for the entire nation based on the spline surface fitting method. NADCON (Dewhurst 1990) in the conterminous United States is stated to be not worse than 0.15 m at the one sigma level. Accordingly, NADCON is appropriate for use in mapping, but better modeling of a local transformation is needed for the accuracy requirements of design and surveying. In this study, we determine whether Helmert transformations can provide survey-levels of accuracy allowing for the possibility of dividing Connecticut into zones. Most hand-held field computers, such as data collectors or survey controllers, allow datum transformations with Helmert transformations, so, if an acceptable Helmert transformation can be found, then its field implementation is straightforward.

As a linear estimation problem, the Helmert transformation can be resolved by the classical least squares method (Tamim and Schaffrin 1995, Tong et al., 2009, 2011) that assumes all the observations are of the same accuracy and only the observations from one datum contains errors. However, geodetic positions in NAD 27 have variability accuracy as indicated by the order of the station (Federal Geodetic Control Committee, 1984). The NGS now reports both network and local (Soler and Smith 2010) accuracies for individual stations. The total least squares (TLS) method, first introduced by Golub and Van Loan (1980), allows errors in the observations from both datums, which conceptually matches the available data. Therefore, we compared ordinary and total least squares, with and without weights.

Methods

Helmert transformations of planimetric positions (eastings and northings) can have a two-dimensional translation vector, a single rotation angle around the projection plane's *z*-axis, and one or two scale factors. This is a maximum of five parameters but only one scale factor is usually used, so two-dimensional Helmert transformations typically have four parameters. Helmert transformations of three-dimensional positions can have a three-dimensional translation vector, a rotation angle around each axis, and up to three scale factors. This is a maximum of nine parameters but only one scale factor is typical, for a total of seven parameters. Helmert transformations for modern reference frames include time-dependent terms for each parameter. The time-dependent terms capture the subtle evolution of the frames over time. No time-dependent information exists for NAD 27 so no time-dependent terms will be included in this study. The mathematical details are in the appendix.

The least squares method has several forms: weighted and non-weighted, and ordinary least squares and total least squares. Non-weighted, ordinary least squares treats all observations as having identical statistical variance. Weighted, ordinary least squares weights more precise observations more heavily than less precise observations. Here, a weighted least squares method was applied to capture any effect due to station-accuracy order. There are three orders of NGS stations in Connecticut. The weights for the first-, second-, and third-order stations were set to 10, 5, and 1 corresponding to the precision of first-, second-, and third-order points being 1: 100 000, 1:50 000, and 1:10 000, respectively (Federal Geodetic Control Committee, 1984).

In this study, the ordinary and weighted least squares methods assume that errors only occur in the NAD 27 coordinates. However, the coordinates in both NAD 83 and NAD 27 have errors so the total least squares method (TLS) is also applied in this study to account for errors in both data sets. Weighted total least squares (WTLS) accounts for errors on both sides of the transformation equation, as well as the accuracy order of the survey markers.

Data Description and Analysis

There are markers with coordinates in both NAD 27 and NAD 83 in the NGS control marker database. In this study, the NGS data are used to parameterize and validate the transformation models. There are 2236 stations with both NAD 27 and NAD 83 coordinates in Connecticut and in a 10-km buffered area in Rhode Island, Massachusetts, and New York. Of these, 1920 points are located in Connecticut. Fifty randomly chosen markers from the Connecticut subset were reserved for validation and were not used to develop transformations. We used the most recent NAD 83 coordinates but did not transform them all to a common realization because the differences among the NAD 83 coordinates ended up being very small compared to the magnitude of our transformations' residuals.

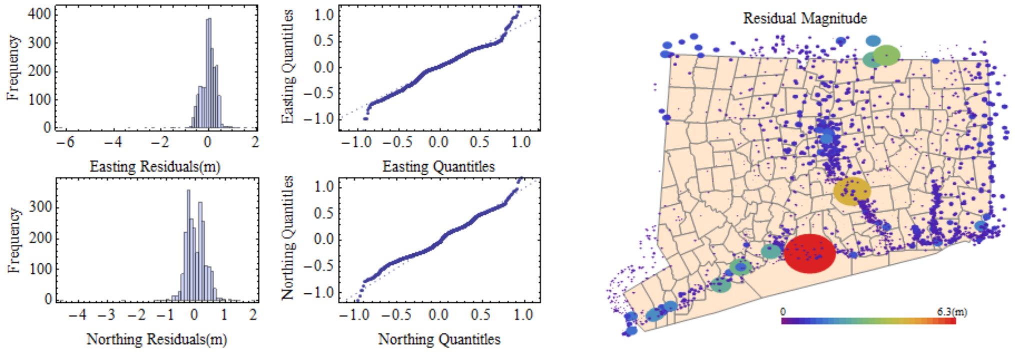
The procedure for creating the proposed transformation has 4 steps: (1) the performance of the transformation was checked by performing the SPC transformation with ordinary least squares method, from which residual maps were used to identify the outliers. Outliers were detected and excluded from the dataset based on the interquartile range. (2) The SPC transformations from NAD 27 coordinates to NAD 83 coordinates were computed with the outliers excluded. Connecticut was subdivided into zones according to spatial residual clustering.

Outliers were detected and excluded again zone-wise. (3) Transformations using weighted least squares and total least squares were computed in both SPC and ECEF coordinate systems. Transformation parameters and error maps are created for each zone. (4) The models were validated using the NGS validation subset and Connecticut Geodetic Survey (CTGS) data.

Results

Overall Performance

The absolute magnitudes of the residuals of the OLS transformation are shown in Fig. 1.b. Shown is a political boundary map of Connecticut overlaid with colored dots whose locations on the map mark the stations' locations and whose colors and radii reflect the residuals' magnitudes.



(a) (b)

Figure 1. The residuals of the SPC transformation using the full dataset.
(a) Histograms and QQ plots of residuals in easting and northing directions.
(b) Residuals' magnitudes (the root sum square of residuals in both easting and northing) both as the dots' radii and their colors.

Most of the residuals are within -1 m and +1 m (Fig. 1a). The easting residuals span −6.29 m to +1.91 m and the northing residuals span -4.55 m to +1.96 m. The residuals outside ±1 m cause the tails in the QQ plots to depart sharply from the normal line. An outlier is an observation that lies outside the overall pattern of a distribution (Moore and McCabe 1999, Renze 2013). Apparent outliers can occur by chance if a sample set is too small to capture a distribution's tails, but outliers are usually thought to indicate measurement error (blunders). We argue in favor of measurement error in spite of there being numerous outliers because NAD 27 positions were determined using triangulation and conventional traversing. These methods are not as accurate a positioning technique as GPS over long base lines. There are 52 stations (36 in easting, 19 in northing, and 3 in both easting and northing) singled out as outliers of which 36 stations are in Connecticut. The outliers are listed in Table 1.

Table 1. The NGS permanent identifier (PID) codes of the outliers. *Directions* indicates whether the station is an outlier in the easting direction, the northing direction, or both. Under the *Note* column, "overall" means these stations were determined to be outliers from analyzing the entire (overall) dataset, and "zone" means these stations were determined to be outliers from analyzing just the stations in some zone.

Directions	Station PID	Note
easting	LX3687, LX3748, LX3901, LX3899, LX3912, LX3805, LX7032, MZ2038, LX4420, LX6189, LX3694, LX3700, LX6247, LX6491, LX5497, LX6067, LX5813, LW3302, LW3301, MZ1572, LW3698, LW3539	Overall (22 Points)
	LX4709, LX4631, LX4677, LX4647, LX4646, LX4660, LX7254, LX6988, LX6840, LX6055, LX7170, LX4978, LX3677, LX3895, LX3915, LX3898, LX3900, LX3893, LX6988, MZ2045, MZ2046, MZ2048, MZ2049, LX4422, LX4421, LX6402, LX6406, LX6299, LX6535, LX7170, LX6378, LW3401, LX5427, LX4801, LX5431, LX5418, LX5840, LX6055, LX5540, LX5251, LX5541, LX5266, LX5431, LX5418, LX5531, LX3699, LX3677	Zone (47 Points)
northing	LX3729, LX3907, LX3899, LX7020, LX7062, LX7061, LX6670, LX6641, LX7023, LX7017, LX7015, LX6860, LX6239, LX6491, LX5339, LX5233, MZ1572	Overall (17 Points)
	LX5084, LX7254, LX7024, LX7021, LX7042, LX7067, LX6784, LX6637, LX6661, LX6652, MZ1846, LX7038, LX6305, LX5979, LX5764, LX3677, LX3895, LX3906, LX3790, LX3795, LX3893, LX3905, LX3675, LX7267, LX7266, LX6988, LX7217, LX7134, LX7133, LX7245, LX7244, LX7274, LX7129, LX7130, LX7153, LX7154, LX7126, LX7159, LX7157, LX7218, LX7219, LX7277, LX7213, LX7207, LX7149, LX7175, LX7273, LX7135, LX6959, LX6530, LX6535, LX7170, LX6447, LX6506, LX4748, LX4768, LW3375, LX5505, LX5878, LX6233, LX5979, LX6190, LX6388, LX6223, LX6219, LX6220, LX6205, LX6268, LX6201, LX6236, LX6202, LX6382, LX5492, LX5505, LX5447, LX5455, LX5466, LX5456, LX5448, LX3677, LX3893, LX3675, LX3663, LX6354, LX6535	Zone (85 Points)
both	LX3899, LX6491, MZ1572	(3 Points)
	LX3677,LX3893,LX3895,LX6535,LX6988,LX7170,LX7254	(7 Points)

The transformation's residuals (signed values, not magnitude) from NAD 27 to NAD 83 excluding the outliers appear in Fig. 2. The residuals are reduced to 0.9 m in the worst case but neither the easting nor northing residuals seem normally distributed. The residuals appear to be grouped spatially. In easting direction, the residuals along the eastern border are relatively high, and transformations for all markers shows westward errors (negative residuals). In the northing direction, the transformation shows larger positive errors in the center of the state along the Connecticut River and negative errors along the coast.

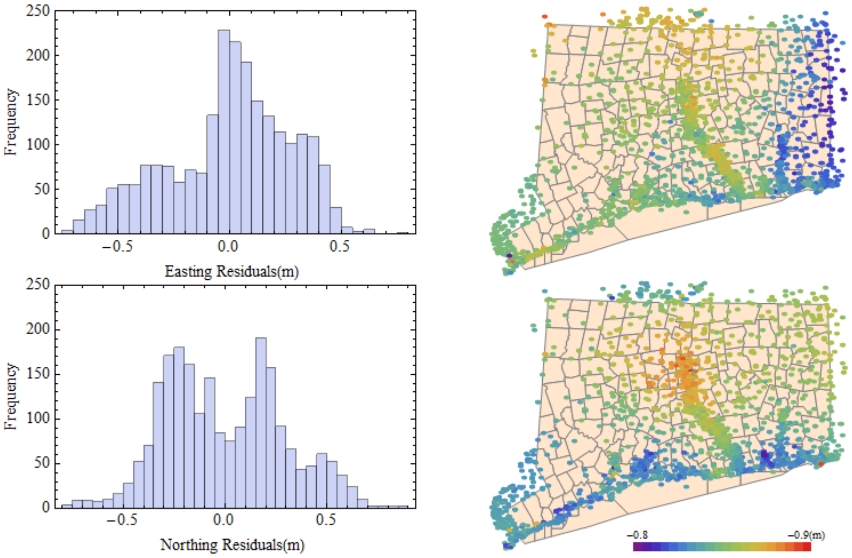


Figure 2. The residual map of the transformation from NAD 27 to NAD 83 using the parameters from ordinary least square for the whole State with outliers excluded.

Zone Definitions

The non-normal residual distributions suggest subdividing Connecticut into eights zones that follow Town boundaries. See Fig. 3. We discovered that better transformations resulted from including markers from 5-km buffers around the zones because the largest residuals occurred in the stations furthest from zones' centers. The residuals of transformations are shown in Fig. 4, in which the overall magnitude of residual are improved from 6.3 m (c.f. Figure 1) to 1.1 m.

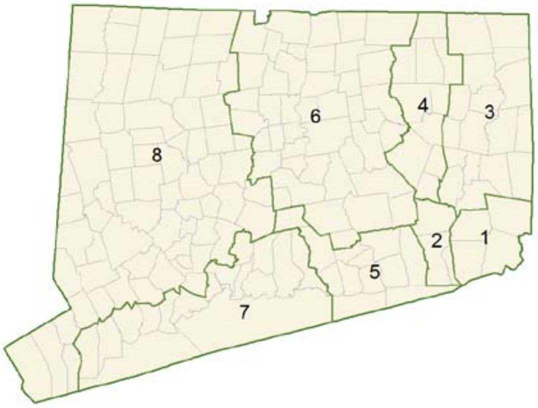


Figure 3. Transformation zone definitions.

The stations with the largest residuals were analyzed in each zone and, using quartile analysis, were identified as outliers and eliminated from the transformation. In total 149 points (132 Points in Connecticut) were identified as outliers (Table 1).

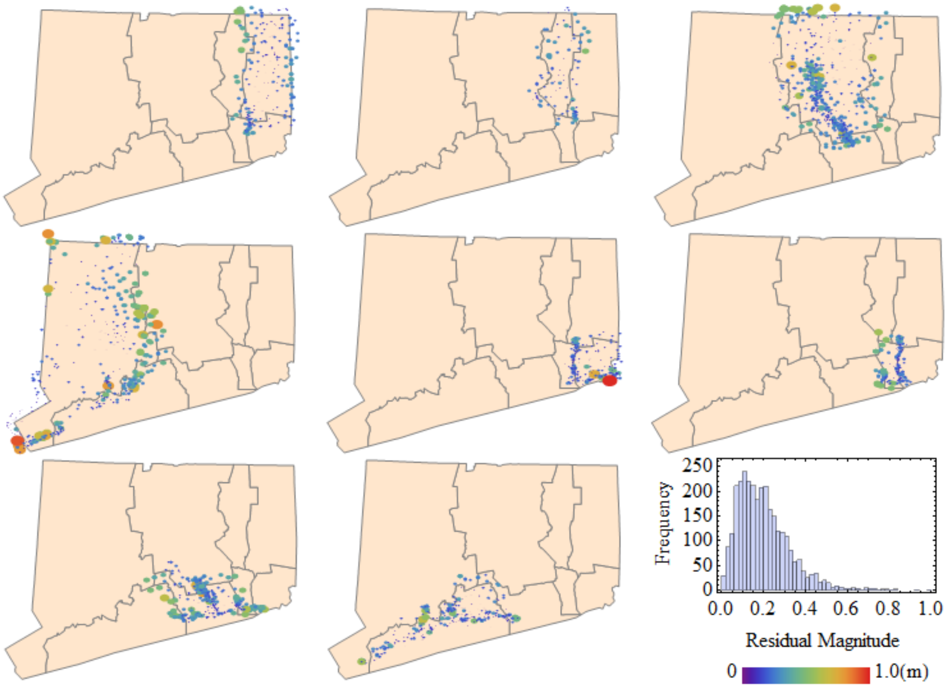
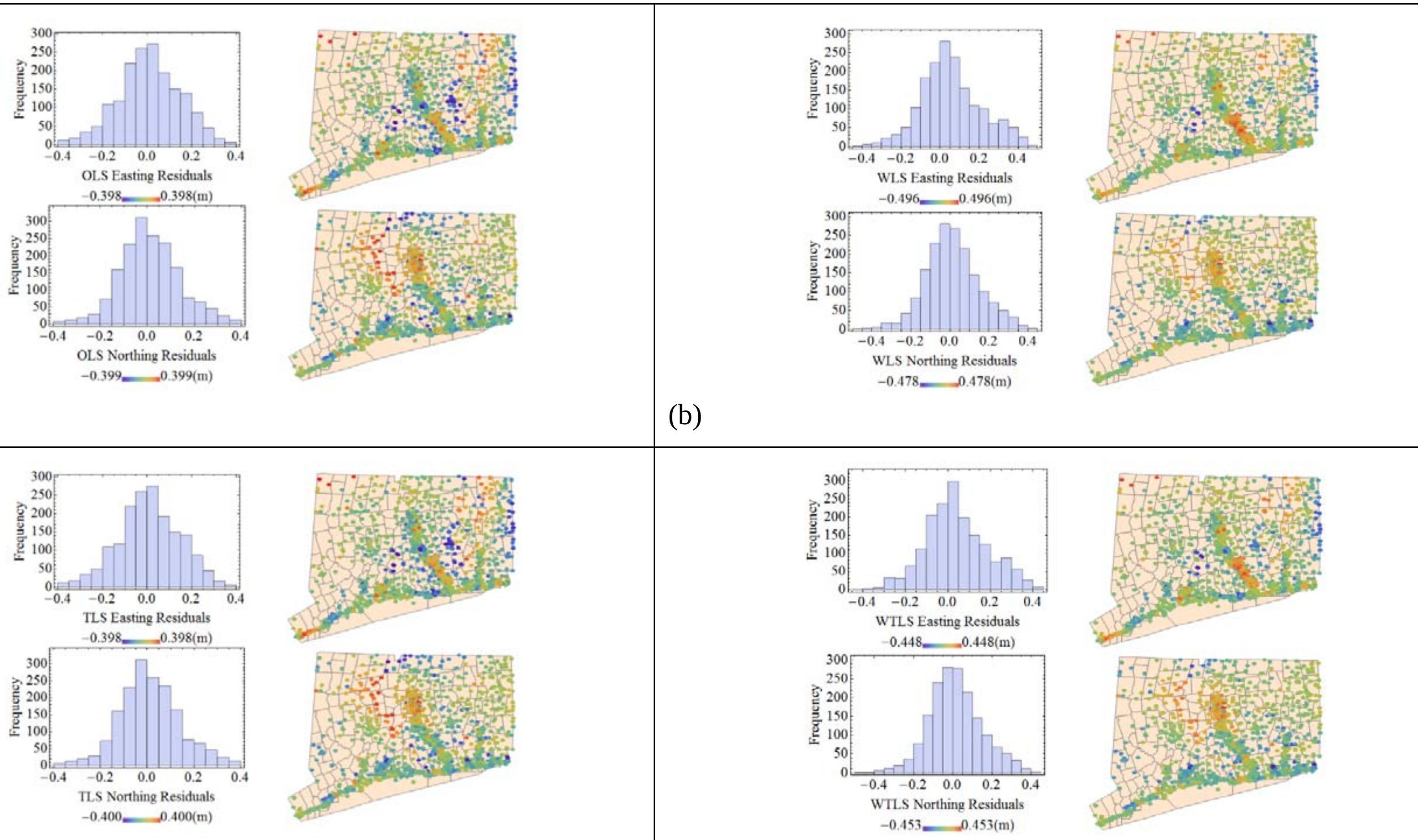


Figure 4. Residual maps of each zone.

Comparing Different Models

Using our zone definitions and with outliers and calibration data excluded, SPC and ECEF models were parameterized using OLS, WOLS, TLS, and WTLS methods. The residual histograms and maps are shown in Fig.s 5 and 6, respectively. The residuals for ECEF model are calculated and transformed to longitude and latitude. The transformations' residuals are improved: they appear more normal, the largest magnitude residual is less than 0.4 m, and the residuals appear more randomly scattered.

None of the four methods nor the two coordinate systems produced a clearly best residual histogram. Therefore, the simplest method – SPC, ordinary least squares – is our pick for the best. The parameters for the eight zones are tabulated in Table 2.



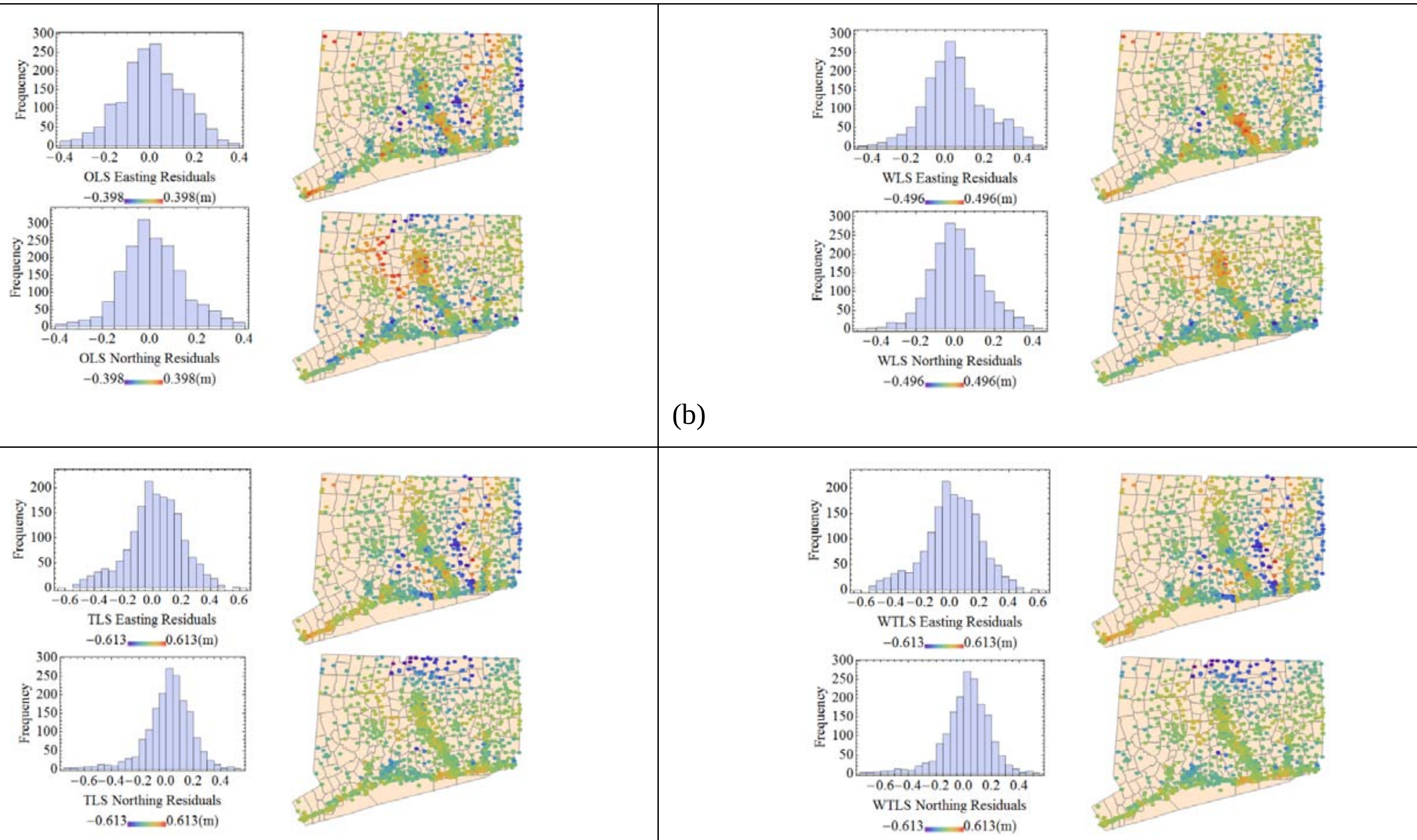
(a)

(b)

(c)

(d)

Figure 5. Histograms and residual maps using the SPC two-dimensional model. Panels show the transformations' residuals histograms using parameters estimated from (a) OLS, (b) WLS, (c) TLS, and (d) WTLS.



(a)

(b)

(c)

(d)

Figure 6. Histograms and residual maps using the ECEF three-dimensional model. Panels show the transformations' residuals histograms using parameters estimated from (a) OLS, (b) WLS, (c) TLS, and (d) WTLS.

Table 2. Transformation parameters (NAD 27 to NAD 83). Parameters from NAD 83 to NAD 27 are obtained simply by negating the sign of the parameter.

ECEF three-dimensional model

Zones	t_x (m)	t_y (m)	t_z (m)	ω_x (mas)	ω_y(mas)	ω_z(mas)	s(ppb)
2	-60.286±13.718	185.669±6.782	142.731±7.109	-522.238±225.775	2760.251±346.162	-4178.321±357.538	-13574.680±979.957
3	-56.166±5.352	172.410±2.123	155.195±2.172	208.094±65.821	501.485±130.191	-2097.611±135.805	-10645.165±299.695
4	-49.203±8.352	153.151±4.281	173.306±4.281	636.146±155.219	-825.858±204.098	-848.639±227.384	-6353.936±626.622
5	-53.859±3.561	170.376±4.969	157.139±5.121	917.433±203.460	-1711.025±109.498	-22.660±115.396	-10132.048±488.118
6	-53.604±2.531	170.653±1.745	157.336±1.475	405.333±57.784	-99.330±61.195	-1491.098±71.488	-10133.680±205.547
7	-46.470±2.801	150.545±2.793	176.646±3.738	87.718±140.052	811.202±88.634	-2291.984±59.932	-5618.397±216.055
8	-52.737±1.690	175.247±0.898	153.592±1.085	205.721±34.839	511.309±44.851	-1964.973±43.027	-11009.776±131.066

SPC two-dimensional model

Zones	t_e (m)	t_n (m)	ω (mas)	s(ppb)
1	121957.207±0.145	152408.163±0.145	3096.319±117.168	-9871.201±568.0469
2	121957.018±0.292	152405.919±0.292	4856.931±247.673	-13570.240±1200.751
3	121956.540±0.099	152409.987±0.099	1699.499±75.728	-10650.555±367.141
4	121957.047±0.192	152412.534±0.192	-171.5920±158.414	-6359.235±768.010
5	121956.346±0.133	152413.171±0.133	-1413.624±123.339	-10126.454±597.965
6	121957.067±0.055	152411.112±0.055	826.702±51.923	-10137.536±251.730
7	121958.128±0.046	152409.739±0.046	2081.099±54.590	-5609.316±264.658
8	121957.283±0.026	152409.959±0.026	1625.053±33.126	-11006.356±160.599

Note: mas=$1/1000 * 1/3600$, ppb=$10\hat{}(-9)$.

Here is a list of which towns are in each zone.

1: Groton, Ledyard North Stonington, Stonington;

2: Montville, New London, Waterford;

3: Griswold, Lisbon, Norwich, Preston, Sprague, Voluntown, Brooklyn, Canterbury, Hampton, Killingly, Plainfield, Pomfret, Putnam, Scotland, Sterling, Thompson, Woodstock;

4: Bozrah, Franklin, Lebanon, Ashford, Chaplin, Eastford, Mansfield, Windham, Union, Willington;

5: East Lyme, Lyme, Old Lyme, Clinton, Deep River, Durham, Essex, Killingworth, Old Saybrook, Westbrook;

6: Colchester, Salem, Andover ,Bolton, Columbia, Coventry, Ellington, Hebron, Somers, Stafford, Tolland, Vernon, Avon, Berlin, Bloomfield, East Granby, East Harford, East Windsor, Enfield, Farmington, Glasstonbury, Granby, Harford, Manchester, Marlborough, New Britain, Newington, Rocky Hill, Simsbury, South Windsor, Suffield, West Hartford, Wethersfield, Windsor , Windsor Locks, Chester, Cromwell, East Haddam, East Hampton, Haddam, Middlefield, Middletown, Portland;

7: Bridgeport, Fairfield, Norwalk, Stratford ,Westport, Branford, East Haven, Guilford, Madison, Milford, New Haven, Northbranford, North Haven, West Haven; and

8: Watertown, Winchester, Woodbury, Ansonia, Beacon Falls, Bethany, Cheshire, Derby, Hamden, Meriden, Middlebury, Naugatuck, Orange, Oxford, Prospect, Seymour, Southbury, Wallingford, Waterbury, Wolcott, Woodbridge.

A reserved subset of NGS data and an independent dataset from the Connecticut Geodetic Survey (CTGS) were used to validate the transformations and estimated parameters. The residuals of the transformation for NGS and CTGS data are shown in Figure 7. The statistical summary of the validation are given in Table 3. For reserved NGS data, the errors of the transformation are no worse than 0.4 meter and there are no obvious clusters patterns. The histograms in Fig. 7 and the statistical summary in Table 3 suggest that the transformation in the northing direction is better than that in easting direction, having means of 0.001 m and 0.040 m, respectively, for the best transformation. For CTGS data, the worst error is less than 0.4 meters.

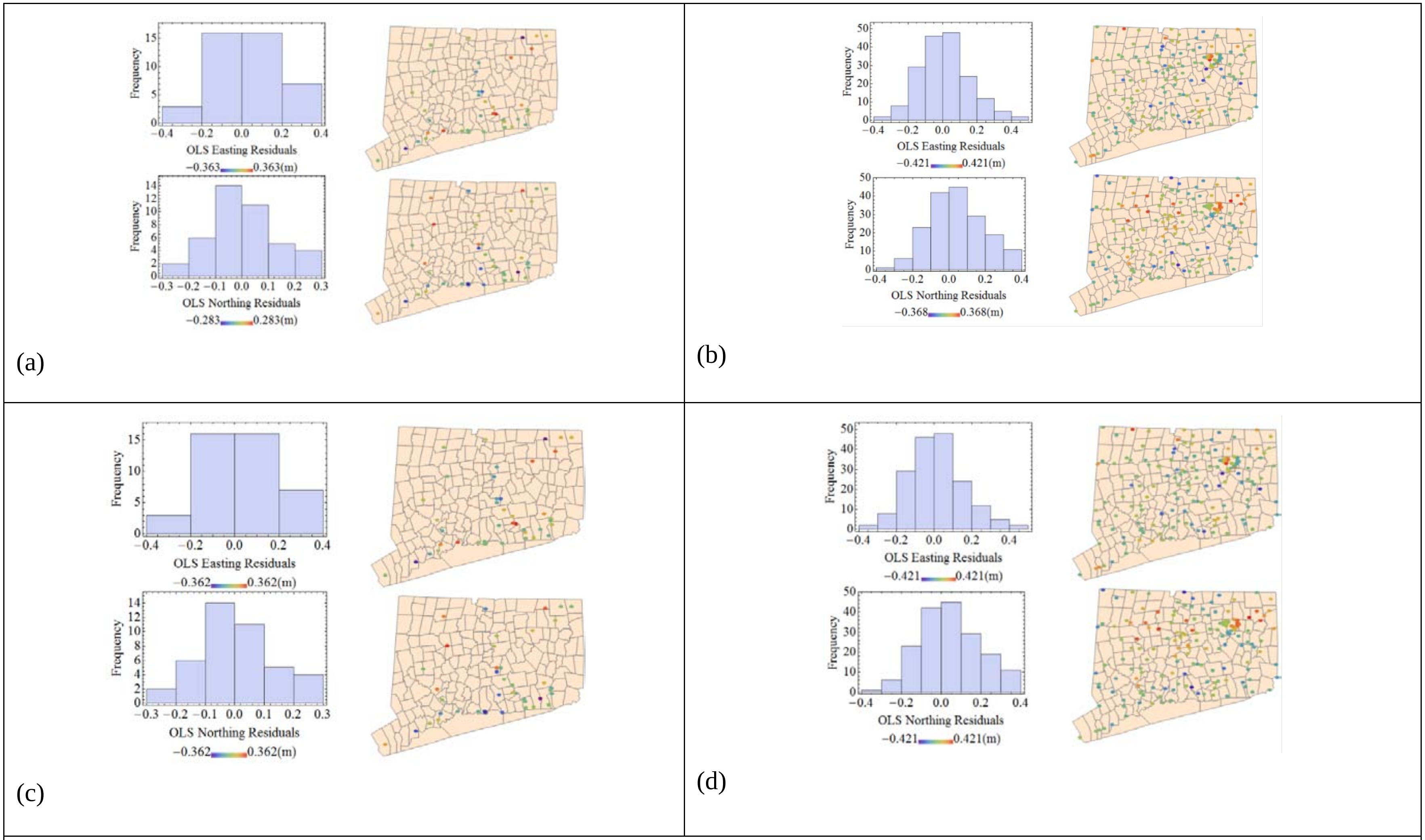


Figure 7. The transformations' residuals histograms from the SPC and ECEF models for both NGS validation subset and CTGS data. Panels (a) and (b) are SPC transformations for NGS validation subset and CTGS data, respectively. Panels (c) and (b) are ECEF transformations for NGS validation subset and CTGS data, respectively.

Table 3. Statistical summary of the validation using the CTGS data (SPC and ECEF)

		NGS Data		CTGS Data	
		easting	northing	easting	northing
Model	Method	Mean(SD)	Mean(SD)	Mean(SD)	Mean(SD)
SPC	OLS	0.040(0.159)	0.001(0.128)	0.016(0.144)	0.045(0.144)
	WOLS	0.079(0.160)	0.004(0.124)	0.042(0.144)	0.044(0.138)
	TLS	0.040(0.159)	0.001(0.128)	0.016(0.144)	0.046(0.144)
	WTLS	0.040(0.157)	0.001(0.123)	0.016(0.144)	0.045(0.139)
ECEF	OLS	−0.040(0.159)	0.001(0.127)	0.016(0.144)	0.045(0.144)
	WOLS	0.078(0.160)	0.004(0.123)	0.042(0.144)	0.044(0.138)
	TLS	0.054(0.225)	−0.017(0.179)	0.047(0.202)	−0.044(0.167)
	WTLS	0.43(0.836)	0.15(0.579)	0.979(0.757)	−0.485(0.822)

Discussion

The residuals in Fig. 1 do not look spatially random. They cluster in magnitude along the edges of water bodies: Long Island Sound along the southern border, the Connecticut River in the central part of the State, and the Thames River to the east. These residuals are also generally larger than stations not interconnected by baselines that cross water bodies. This is sensible because it is well known that observations across water bodies are affected by varying amounts of refraction that is difficult to correct, and, for NAD 27, electronic distance measurement (EDM) instruments were not yet available and only triangulation methods could be used. Also, the stations along Long Island Sound were not controlled to the south, which would generally be expected to weaken the surveying network in that area.

The residuals in Fig. 2, coded by color only (not radius), show general patterns that reflect geography, as noted above, but also geodesy. We calculated a significant scale parameter – roughly 10 000 ppb – for the Helmert transformation. The scale parameter's effect is to move all places towards the center of the State, and this is reflected, somewhat weakly, as a blue shift in the east and a red shift in the west in the eastings, and a blue shift in the south and a red shift to the north in the northings. The northing residuals are bimodal (Fig. 2 a). The stations have clustered residuals in the center of the State around Hartford and also along the coast. Note the paucity of stations in Connecticut's western highlands along the border with New York. We extrapolate that, if there were many more western stations, that the easting residuals would appear bimodal as well. These multi-modal histograms and the spatial clustering of the residuals justify subdividing Connecticut into zones with the goal of determining transformations whose residuals are normally distributed. Figure 4 shows the results, and the spatial distribution of the residuals appears more random.

The final overall results appear in Fig.s 5 and 6. The zones create the desired randomizing of the residuals and the overall histograms look reasonably normal. No method is clearly best; likewise, neither coordinate system. Without a winner by performance, we picked the simplest model: ordinary least squares in SPC coordinates. We also provide the ECEF parameters in case they are more convenient. (Some data collectors might not implement the four-parameter transformation.) The inverse transformation (NAD 83 to NAD 27) comes by simply negating the arithmetic signs of the parameters.

We were surprised that weighted total least squares was not the best; in fact, it was arguably the worst. However, although NAD 83 coordinates certainly have error, they also should have much less error than the NAD 27 equivalents due to older measurement techniques. This large disparity in the weights tends to minimize the added functionality of total least squares, so the result is reasonable in hindsight.

Conclusions

The analyses in this study enable the self-inconsistencies of NAD 27 to be seen. The mathematics provide the means to establish what the NAD 27 coordinates of the markers should have been had they been positioned with methods as accurate as GPS, and the conclusion based on the transformation residuals is that NAD 27 coordinates are, in retrospect, too inaccurate for a Helmert transformation to capture the unpredictable distortions in the coordinates. This distortion was known *a priori* for NAD 27 on the whole, for the entire nation. Triangulation and traditional traversing have well understood error propagation characteristics that prohibit GPS-accuracy levels across very large areas, like across North America. With GPS the distance between a marker on the west coast and a marker on the east can be established with centimeter accuracy, which is impossible with optomechanical methods. What was unknown was whether NAD 27 coordinates might be accurate enough over quite small areas so that a transformation would be successful – after all, the traditional methods are highly accurate across relatively small areas. However, even subdividing Connecticut into eight zones was unsuccessful, and eight was the smallest possible because further subdivision would reduce the number of stations per zone to be too low for the statistics to work properly.

This study showed that the self-inconsistencies of NAD 27 preclude any realistic hope of developing a survey-accuracy transformation regardless of the mathematical approach. Still, there is a benefit to knowing that only a mapping/GIS quality transformation can be achieved. This fact alone allows CTDOT engineering management and land surveying professionals to know that there is no panacea to the dilemma. There is no need to spend further monies pursuing the simplest solution. The authors, therefore, conclude that, for project-level control at design/construction required accuracies, only a land surveyor can deliver the required control survey results.

Now, with the release of NAD 83(2011), Connecticut is facing a troubling situation for its geodetic control. Connecticut positioning professionals are confronted with control coordinates that span datums from NAD 27 through NAD 83(2011) compounded by the certain knowledge that an entirely new national reference frame will be realized within the decade (per the NGS 10-year plan). The disparate accuracy levels and general confusion that inevitably arises from mixing coordinates from disparate datums is our reality. This suggests a strategy should be developed to reorganize our geodetic control assets.

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Appendix

This appendix details the mathematics used to determine the Helmert transformations. Both the two-dimensional and three-dimensional Helmert transformations can be written as:

$$\boldsymbol{P}_{83} = (1 - s)\boldsymbol{R} \cdot \boldsymbol{P}_{27} + \boldsymbol{T} \tag{1}$$

where $\boldsymbol{P}_{83}$ is a position in NAD 83; it is $[\mathrm{x}_{83} \quad \mathrm{y}_{83}]^{\mathrm{T}}$ in SPC coordinates and $[\mathrm{x}_{83} \quad \mathrm{y}_{83} \quad \mathrm{z}_{83}]^{\mathrm{T}}$ in the ECEF coordinates, where $[.]^{T}$is the transpose operator. Similarly, $\boldsymbol{P}_{27}$ is a position in NAD 27 and it is $[\mathrm{x}_{27} \quad \mathrm{y}_{27}]^{\mathrm{T}}$ in SPC coordinates and $[\mathrm{x}_{27} \quad \mathrm{y}_{27} \quad \mathrm{z}_{27}]^{\mathrm{T}}$ in ECEF coordinates. All coordinates have linear units of meters. The scale factor s (ppb) is a difference from unity. The rotation matrix $\boldsymbol{R}$ aligns the datums' coordinate axes. Well-defined datums will generally agree about the orientation of their coordinate axes, so the angles in $\boldsymbol{R}$ are usually quite small. The sine of a small angle approximately equals the angle, and the cosine of a small angle approximately equals one so the rotation matrix can be written as $\boldsymbol{R} = \begin{bmatrix} 1 & \omega \\ -\omega & 1 \end{bmatrix}$ and $\boldsymbol{R} = \begin{bmatrix} 1 & \omega_z & -\omega_y \\ -\omega_z & 1 & \omega_x \\ \omega_y & -\omega_x & 1 \end{bmatrix}$ for the SPC and ECEF models, respectively. The translation vector $\boldsymbol{T}$ is the coordinates of NAD 83 origin in NAD 27. For the SPC model $\boldsymbol{T} = [\mathrm{t}_{\mathrm{x}} \quad \mathrm{t}_{\mathrm{y}}]^{\mathrm{T}}$and for the ECEF model $\boldsymbol{T} = [\mathrm{t}_{\mathrm{x}} \quad \mathrm{t}_{\mathrm{y}} \quad \mathrm{t}_{\mathrm{z}}]^{\mathrm{T}}$.

Parameterization

To estimate the ECEF-model's parameters using the ordinary least square method, Eq. (1) is rewritten as:

$$\begin{bmatrix} x_{83} \\ y_{83} \\ z_{83} \end{bmatrix} = (1 - s) \begin{bmatrix} 1 & \omega_z & -\omega_y \\ -\omega_z & 1 & \omega_x \\ \omega_y & -\omega_x & 1 \end{bmatrix} \begin{bmatrix} x_{27} \\ y_{27} \\ z_{27} \end{bmatrix} + \boldsymbol{T} \tag{2}$$

After combining known information (coordinates) as their differences on the left and neglecting higher-order $s{\cdot}\omega_i$ terms, Eq. (2) can be written:

$$\begin{bmatrix} \Delta\mathrm{x} \\ \Delta\mathrm{y} \\ \Delta\mathrm{z} \end{bmatrix}_{83-27} = \begin{bmatrix} 1 & 0 & 0 & 0 & -z_{27} & y_{27} & -x_{27} \\ 0 & 1 & 0 & z_{27} & 0 & -x_{27} & -y_{27} \\ 0 & 0 & 1 & -y_{27} & x_{27} & 0 & -z_{27} \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \\ \omega_x \\ \omega_y \\ \omega_z \\ s \end{bmatrix} \tag{3}$$

Similarly, the SPC model is written:

$$\begin{bmatrix} \Delta\mathrm{x} \\ \Delta\mathrm{y} \end{bmatrix}_{83-27} = \begin{bmatrix} 1 & 0 & -y_{27} & -x_{27} \\ 0 & 1 & x_{27} & -y_{27} \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ \omega \\ s \end{bmatrix} \tag{4}$$

In matrix form, after swapping both sides of the equal sign, both the SPC and ECEF models are written:

$$\boldsymbol{A}_{3n\times k}\hat{\boldsymbol{\beta}}_{k\times 1} = \boldsymbol{B}_{3n\times 1} \tag{5}$$

where k = 3 or 7 for the SPC model and the ECEF model, respectively, and n is the number of stations.

There are four parameters to estimate for the SPC model: $\hat{\boldsymbol{\beta}}_{\mathbf{4}} = [t_x \quad t_x \quad \omega \quad s]$.

There are seven parameters to estimate for the ECEF model:

$\hat{\boldsymbol{\beta}}_{\mathbf{7}} = [t_x \quad t_x \quad t_z \quad \omega_x \quad \omega_y \quad \omega_z \quad s]$.

A weighted least squares method was also applied to capture any effect due to station-accuracy order. There are three orders of NGS stations in Connecticut. The weights for the first-, second-, and third-order stations were set to 10, 5, and 1 corresponding to the precision of first-, second-, and third-order points being 1: 100 000, 1:50 000, and 1:10 000, respectively (Federal Geodetic Control Committee, 1984). Assuming independence, the weight matrix is $3n\times 3n$ diagonal, with the diagonal elements being the weights of the corresponding station, so Eq. (5) is augmented with a weight matrix and becomes:

$$(\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{A})\hat{\boldsymbol{\beta}} = \boldsymbol{A}^T\boldsymbol{W}\boldsymbol{B} \tag{6}$$

The ordinary and weighted least squares methods assume that errors only occur in $\boldsymbol{B}$. However, the coordinates in both NAD 83 and NAD 27 have errors so the total least squares method is also applied in this study to account for errors in both data sets.

The most common TLS algorithm is based on singular value decomposition (Van Huffel and Vandewalle 1991; Han 2010), and several weighing mechanisms have been developed (Tong et al., 2011). In the TLS method, we seek to find $\hat{\beta}$ that minimizes error matrices $\boldsymbol{E}$ and $\boldsymbol{F}$ for $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively.

$$(\boldsymbol{A}+\boldsymbol{E})\boldsymbol{\beta} = \boldsymbol{B}+\boldsymbol{F} \tag{7}$$

$$[\boldsymbol{A}+\boldsymbol{E} \quad \boldsymbol{B}+\boldsymbol{F}]\left[\begin{matrix}\boldsymbol{\beta}\\ -1\end{matrix}\right] = 0 \tag{8}$$

The solution of Eq. (7) requires the singular value decomposition ($\boldsymbol{U}$, $\sum$, $\boldsymbol{V}$) of the augmented matrix$[\boldsymbol{A} \quad \boldsymbol{B}]$. The estimated parameters are:

$$\hat{\boldsymbol{\beta}} = -\boldsymbol{V}_{AB}\boldsymbol{V}_{BB}^{-1} \tag{9}$$

where $\boldsymbol{V}_{\mathrm{AB}}$ and $\boldsymbol{V}_{\mathrm{BB}}$ are the blocks of $\boldsymbol{V}$ partitioned corresponding to the dimensions of $\boldsymbol{A}$ and $\boldsymbol{B}$, i.e. $\boldsymbol{V}_{\mathrm{AA}}$ is $k\times 1$ and $\boldsymbol{V}_{\mathrm{BB}}$ is 1×1. This solution holds if $\boldsymbol{V}_{\mathrm{BB}}$ is nonsingular, which means, in this case, if $\boldsymbol{V}_{\mathrm{BB}}$ does not equal zero.

Weighted total least squares (WTLS) accounts for errors on both sides of the transformation equation, as well as the accuracy order of the survey markers. The algorithm follows an iterative process:

1. Obtain $\hat{\beta}^{(0)}$ from Eq. (9).
2. Calculate $\hat{\lambda}^{(i)}$and $\hat{\beta}^{(i+1)}$

$$\hat{\boldsymbol{\lambda}}^{(i)} = \{\boldsymbol{Q}_B + (\{[\hat{\boldsymbol{\beta}}^{(i)}]^T \otimes \boldsymbol{I}_{3n}\}\boldsymbol{Q}_A[\hat{\boldsymbol{\beta}}^{(i)} \otimes \boldsymbol{I}_{3n}])\}^{-1}[\boldsymbol{B} - \boldsymbol{A}\hat{\boldsymbol{\beta}}^{(i)}] \tag{10}$$

$$\begin{aligned}\hat{\boldsymbol{\beta}}^{(i+1)} = {} & (\boldsymbol{A}^T\{\boldsymbol{Q}_B + (\{[\hat{\boldsymbol{\beta}}^{(i)}]^T \otimes \boldsymbol{I}_{3n}\}\boldsymbol{Q}_A[\hat{\boldsymbol{\beta}}^{(i)} \otimes \boldsymbol{I}_{3n}])\}^{-1}\boldsymbol{A})^{-1} \\ & \cdot \{\mathrm{vec}^{-1}(\boldsymbol{Q}_A[\hat{\boldsymbol{\beta}}^{(i)} \otimes \boldsymbol{I}_{3n}]\hat{\boldsymbol{\lambda}}^{(i)})^T\hat{\boldsymbol{\lambda}}^{(i)} \\ & + \boldsymbol{A}^T\{\boldsymbol{Q}_B + (\{[\hat{\boldsymbol{\beta}}^{(i)}]^T \otimes \boldsymbol{I}_{3n}\}\boldsymbol{Q}_A[\hat{\boldsymbol{\beta}}^{(i)} \otimes \boldsymbol{I}_{3n}])\}^{-1}\boldsymbol{B}\}\end{aligned} \tag{11}$$

where $0 \leq i < \infty$ denotes the iteration step, $\boldsymbol{Q}_A$ and $\boldsymbol{Q}_B$ are the inverses of the weight matrices for left- and right-hand sides of Eq. (4), $\otimes$ denotes the operation of Kronecker product of matrices, and vec^{-1} denotes the inverse operation of vectorization, *i.e.* to reconstruct a vector as a matrix.

3. Repeat Step 2 until $\left\|\hat{\boldsymbol{\beta}}^{(\boldsymbol{i}+\mathbf{1})} - \hat{\boldsymbol{\beta}}^{(\boldsymbol{i})}\right\| < \varepsilon$, where $\|.\|$ is the Frobenius norm and ε is a given a threshold value of 10^{-5}.
4. The parameter estimate is $\widehat{\boldsymbol{\beta}}^{(\boldsymbol{i}+\mathbf{1})}$.