8.6 Design Equations

8.6.1 General

An exact theoretical analysis of culvert flow is extremely complex because the following is required:

- analyzing nonuniform flow with regions of both gradually varying and rapidly varying flow
- determining how the flow type changes as the flow rate and tailwater elevations change
- applying backwater and drawdown calculations, energy and momentum balance
- applying the results of hydraulic model studies
- determining if hydraulic jumps occur and if they are inside or downstream of the culvert barrel

8.6.2 Approach

The procedures in this chapter use the following:

Control Section: The control section is the location where there is a defined relationship between the flow rate and the upstream water surface elevation.

- Inlet control is governed by the inlet geometry.
- Outlet control is governed by a combination of the culvert inlet geometry, the barrel characteristics and the tailwater.

Minimum Performance: Minimum performance is assumed by analyzing both inlet and outlet control and using the highest headwater. The culvert may operate more efficiently at times (more flow for a given headwater level), but it will not operate at a lower level of performance than calculated.

8.6.3 Inlet Control

For inlet control, the control section is at the upstream end of the barrel (the inlet). The flow passes through critical depth near the inlet and becomes shallow, high velocity (supercritical) flow in the culvert barrel. Depending on the tailwater, a hydraulic jump may occur downstream of the inlet.

Headwater Factors

Headwater depth is measured from the inlet invert or the channel invert (if the culvert is buried for fish passage) of the inlet control section to the surface of the upstream pool. This measurement should be taken some distance upstream of the culvert entrance to avoid the “drawdown” curve.

Inlet area is the cross-sectional area of the face of the culvert. Generally, the inlet face area is the same as the barrel area.

Inlet edge configuration describes the entrance type. Some typical inlet edge configurations are thin edge projecting, mitered, square edges in a headwall and beveled edge.

Inlet shape is usually the same as the shape of the culvert barrel. Typical shapes are rectangular, circular, elliptical and arch. Check for an additional control section, if different than the barrel.

Hydraulics

Three regions of flow are shown in the Figure 8-1: unsubmerged, transition and submerged:

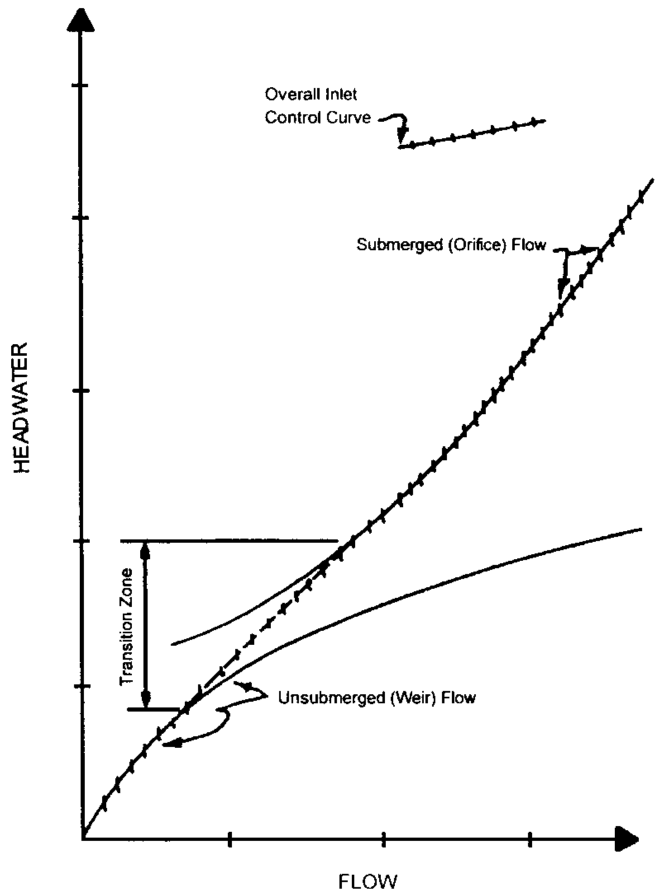

Figure 8-1 Unsubmerged, Transition And Submerged

Unsubmerged

For headwater below the inlet crown, the entrance operates as a weir.

- A weir is a flow control section where the upstream water surface elevation can be predicted for a given flow rate.
- The relationship between flow and water surface elevation must be determined by model tests of the weir geometry or by measuring prototype discharges.
- These tests are then used to develop equations. Appendix A of HDS5 contains the equations which were developed from model test data, see Figure 8-2, Flow Type I:

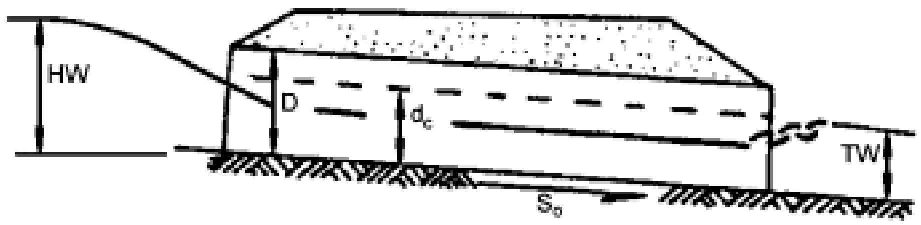

Figure 8-2 Flow Type I

Submerged

For headwaters above the inlet crown, the culvert operates as an orifice.

- An orifice is an opening, submerged on the upstream side and flowing freely on the downstream side, which functions as a control section.
- The relationship between flow and headwater can be defined based on results from model tests. Appendix A of HDS 5 contains flow equations which were developed from model test data. See Figure 8-3 flow type V.

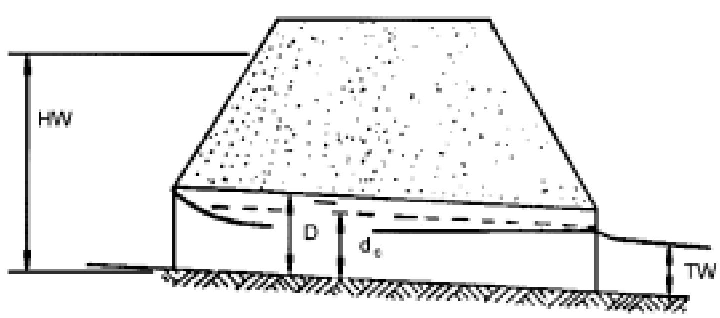

Figure 8-3 Flow Type V

Transition Zone: The transition zone is located between the unsubmerged and the submerged flow conditions where the flow is poorly defined. This zone is approximated by plotting the unsubmerged and submerged flow equations and connecting them with a line tangent to both curves.

Nomographs: The inlet control flow versus headwater curves which are established using the above procedure are the basis for constructing the inlet control design nomographs. Note that in the inlet control nomographs, HW is measured to the total upstream energy grade line including the approach velocity head.

8.6.4 Outlet Control

Outlet control has depths and velocities which are subcritical. The control of the flow is at the downstream end of the culvert (the outlet). The tailwater depth is either assumed to be critical depth near the culvert outlet or the downstream channel depth, whichever is higher. In a given culvert, the type of flow is dependent on all of the barrel factors. All of the inlet control factors also influence culverts in outlet control.

Barrel Roughness: Barrel roughness is a function of the material used to fabricate the barrel. Typical materials include concrete and corrugated metal. The roughness is represented by a hydraulic resistance coefficient such as the Manning n value. Typical Manning n values are presented in Appendix A.

Barrel Area: Barrel area is measured perpendicular to the flow.

Barrel Length: Barrel length is the total culvert length from the entrance crown to the exit crown of the culvert. Because the design height of the barrel and the slope influence the actual length, an approximation of barrel length is usually necessary to begin the design process.

Barrel Slope: Barrel slope is the actual slope of the culvert barrel, and is often the same as the natural stream slope. However, when the culvert inlet or outlet is raised or lowered, the barrel slope is different from the stream slope.

Tailwater Elevation: Tailwater is based on the downstream water surface elevation. Backwater calculations from a downstream control, a normal depth approximation, or field observations are used to define the tailwater elevation (see Section 8.3.5 and 8.3.6).

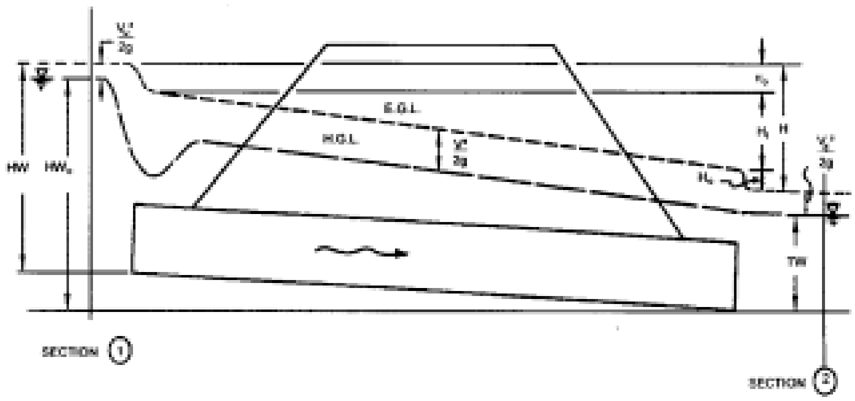

Figure 8-4 Flow Type IV

8.6.5 Hydraulics (Outlet Control)

Full flow in the culvert barrel is assumed for the analysis of outlet control hydraulics. Outlet control flow conditions can be calculated based on an energy balance from the tailwater pool to the headwater pool.

Losses $\mathbf{H_L = H_E + H_f + H_v + H_b + H_j + H_g}$ **(8.1)**

Where: H_L = total energy loss, m (ft)
H_E = entrance loss, m (ft)
H_f = friction losses, m (ft)
H_v = exit loss (equivalent to H_o (see equation 8.4d) m (ft))
H_b = bend losses, m (ft) (see HDS 5)
H_j = losses at junctions, m (ft) (see HDS 5)
H_g = losses at grates, m (ft) (see HDS 5)

Velocity $\mathbf{V = Q/A}$ **(8.2)**

Where: V = average barrel velocity, m/s (ft/s)
Q = flow rate, m^3/s (ft^3/s)
A = cross sectional area of flow with the barrel full, m^2 (ft^2)

Velocity head $\mathbf{H_v = V^2/2g}$ **(8.3)**

Where: g = acceleration due to gravity, 9.81 m/s^2 (32.2 ft/s^2)

Entrance loss $\mathbf{H_E = K_E (V^2/2g)}$ **(8.4a)**

Where: K_E = entrance loss coefficient, See Appendix B

Friction loss **$H_f = [(19.63n^2L)/R^{1.33}]\ [V^2/2g]$ ($H_f = [(29n^2L)/R^{1.33}]\ [V^2/2g]$)** **(8.4b)**

Where: n = Manning's roughness coefficient (See Appendix A)
L = length of the culvert barrel, m (ft)
R = hydraulic radius of the full culvert barrel = A/P, m (ft)
P = wetted perimeter of the barrel, m (ft)

Exit loss **$H_o = 1.0\ [(V^2/2g) - (V_d^2/2g)]$** **(8.4c)**

Where: V_d = channel velocity downstream of the culvert, m/s (ft/s) (usually neglected, see equation 8.4d).

$H_o = H_v = V^2/2g$ **(8.4d)**

Barrel losses **$H = H_E + H_o + H_f$**

$H = [1 + K_e + (19.63n^2L/R^{1.33})]\ [V^2/2g]$ ($H = [1 + K_e + (29n^2L/R^{1.33})]\ [V^2/2g]$) **(8.5)**

Energy Grade Line: The energy grade line represents the total energy at any point along the culvert barrel. Equating the total energy at sections 1 and 2, upstream and downstream of the culvert barrel in Figure 8-4, the following relationship results:

$HW_o + (V_u^2/2g) = TW + (V_d^2/2g) + H_L$ **(8.6)**

Where: HW_o = headwater depth above the outlet invert, m (ft)
V_u = approach velocity, m/s (ft/s)
TW = tailwater depth above the outlet invert, m (ft)
V_d = downstream velocity, m/s (ft/s)
H_L = sum of all losses (equation 8.1)

Hydraulic Grade Line: The hydraulic grade line is the depth to which water would rise in vertical tubes connected to the sides of the culvert barrel. In full flow, the energy grade line and the hydraulic grade line are parallel lines separated by the velocity head except at the inlet and the outlet.

Nomographs (full flow): The nomographs were developed assuming that the culvert barrel is flowing full and:

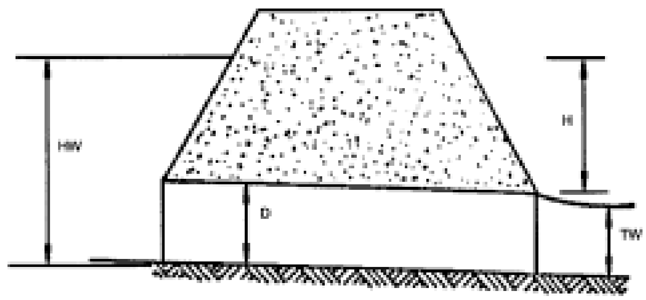

Figure 8-5 Flow Type VI

- $TW \geq D$, Flow Type IV (see Figure 8-4) or
- $d_c \geq D$, Flow Type VI (see Figure 8-5)
- V_u is small and its velocity head can be considered to be a part of the available headwater (HW) used to convey the flow through the culvert.
- V_d is small and its velocity head can be neglected.
- Equation (8.6) becomes:

$$\mathbf{HW = TW + H - S_oL} \qquad \mathbf{(8.7)}$$

Where: HW = depth from the inlet invert to the energy grade line, m (ft)
H = is the value read from the nomographs (equation 8.5), m (ft)
S_oL = drop from inlet to outlet invert, m (ft)

Nomographs (Partly full flow): Equations (8.1) through (8.7) were developed for full barrel flow. The equations also apply to the flow situations which are effectively full flow conditions, if $TW < d_c$, Figure 8-6.

Backwater calculations may be required which begin at the downstream water surface and proceed upstream. If the depth intersects the top of the barrel, full flow extends from that point upstream to the culvert entrance.

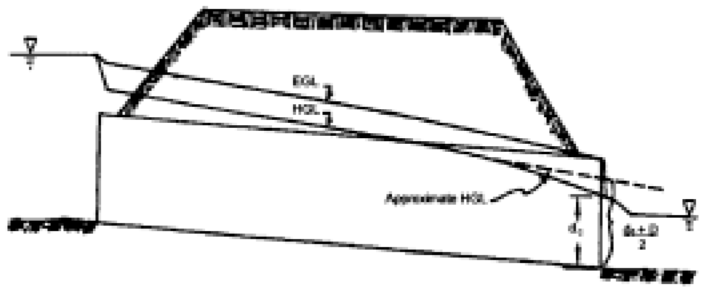

Figure 8-6 Flow Type VII

Nomographs (Partly full flow) – Approximate method: Based on numerous backwater calculations performed by the FHWA, it was found that the hydraulic grade line pierces the plane of the culvert outlet at a point approximately one-half way between critical depth and the top of the barrel or $(d_c + D)/2$ above the outlet invert. TW should be used if higher than $(d_c + D)/2$. The following equation should be used:

$$\mathbf{HW = h_o + H - S_oL} \qquad \mathbf{(8.8)}$$

Where: h_o = the larger of TW or $(d_c + D)/2$, m (ft)

The approximate method renders adequate results for a HW > 0.75D. For lower headwaters, backwater calculations are required.

(See Figure 8-7 if $TW < d_c$ and Figure 8-8 if $TW > d_c$)

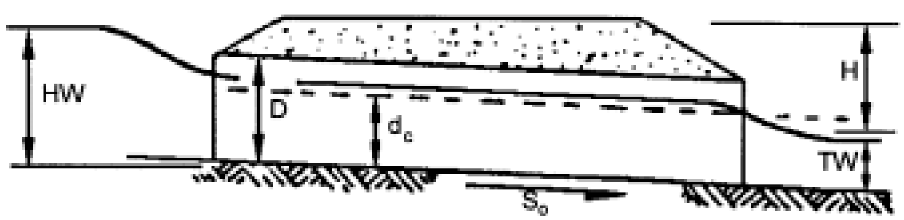

Figure 8-7 Flow Type II, $TW < d_c$

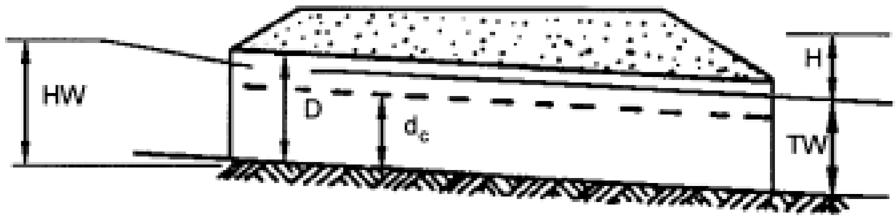

Figure 8-8 Flow Type III, TW > d_c

8.6.6 Outlet Velocity

Culvert outlet velocities shall be calculated to determine need for erosion protection at the culvert exit. Culverts usually result in outlet velocities which are higher than the natural stream velocities. These outlet velocities may require flow readjustment or energy dissipation to prevent downstream erosion.

Inlet Control: The velocity is calculated from equation 8.2 after determining the outlet depth. Either of the following methods may be used to determine the outlet depth.

- Calculate the water surface profile through the culvert. Begin the computation at d_c at the entrance and proceed downstream to the exit. Determine at the exit the depth and flow area.
- Assume normal depth and velocity. This approximation may be used since the water surface profile converges towards normal depth if the culvert is of adequate length. This outlet velocity may be slightly higher than the actual velocity at the outlet. Normal depths in feet may be obtained from design aids in publications such as HDS 3.

Outlet Control: The cross sectional area of the flow is defined by the geometry of the outlet and either critical depth, tailwater depth, or the height of the conduit as defined below:

- Critical depth is used when the tailwater is less than critical depth.
- Tailwater depth is used when tailwater is greater than critical depth, but below the top of the barrel.
- The total barrel area is used when the tailwater exceeds the top of the barrel.