10.7   Stage-Storage Relationship

10.7.1 General

A stage-storage relationship defines the relationship between the depth of water and storage volume in the storage facility. The volume of storage can be calculated by using simple geometric formulas expressed as a function of storage depth. This relationship between storage volume and depth defines the stage-storage curve. A typical stage-storage curve is illustrated in Figure 10-5. After the required storage has been estimated, the configuration of the storage basin must be determined so that the stage-storage curve can be developed. The following relationships can be used for computing the volumes at specific depths of geometric shapes commonly used in detention facilities.
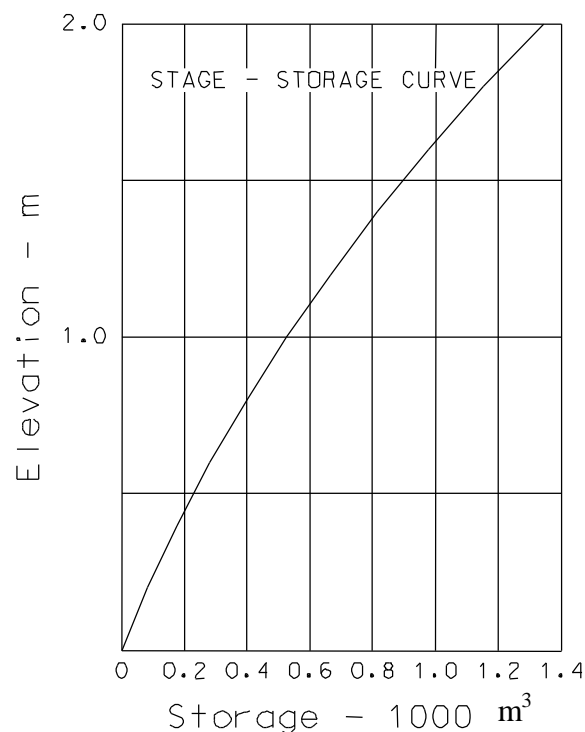
Figure 10-5 Stage-storage Curve

10.7.2 Rectangular Basins

Underground storage tanks are often rectangular. The volume of a rectangular basin can be computed by dividing the volume into triangular and rectangular shapes and using equation 10.6. The variables in equation 10.6 are illustrated in Figure 10-6.

Box: Triangle:

$$V = L\,W\,D \qquad\qquad\qquad\qquad\qquad V = \frac{1}{2}\,W\,\frac{D^2}{S} \qquad\qquad\qquad \textbf{(10.6)}$$

where: V = volume at a specific depth, m^3 (ft^3)
         D = depth of ponding for that shape, m (ft)
         W = width of basin at base, m (ft)
         L = length of basin at base, m (ft)
         S = slope of basin, m/m (ft/ft)

If the basin is not on a slope, then the geometry will consist only of rectangular shaped boxes.
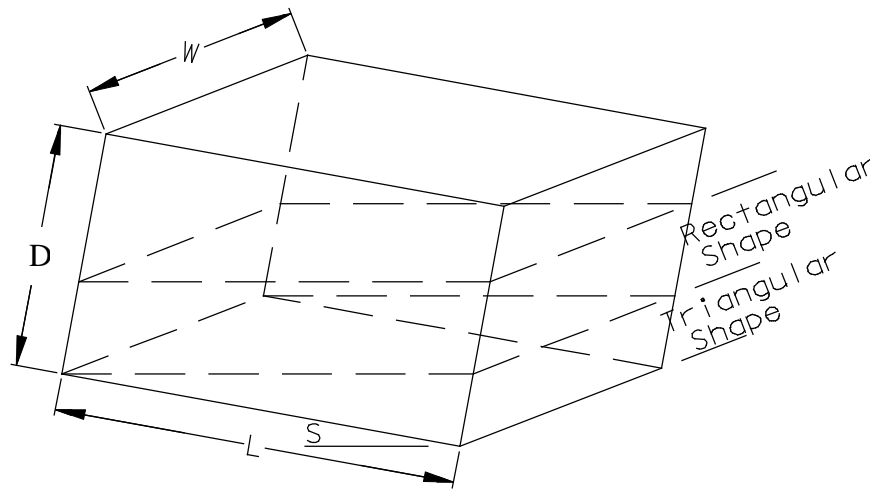

Figure 10-6 Rectangular Basin

10.7.3   Trapezoidal Basins

The volume of a trapezoidal basin can be calculated in a manner similar to that of rectangular basins by dividing the volume into triangular and rectangular shaped components and applying equation 10.7. The variables in equation 10.7 are illustrated in Figure 10-7. "Z" in this equation is the ratio of the horizontal to vertical components of the side slope. For example, if the side slope is 1 to 2 (V:H), "Z" will be equal to 2.

$$V = L\,W\,D + (L + W)\,Z\,D^2 + \frac{4}{3}\,Z^2\,D^3 \qquad\qquad \textbf{(10.7)}$$

where: V = volume at a specific depth, m^3 (ft^3)
         D = depth of ponding or basin, m (ft)
         L = length of basin at base, m (ft)
         W = width of basin at base, m (ft)
         r = ratio of width to length of basin at the base
         Z = side slope factor; ratio of horizontal to vertical components of side slope

Estimating the trial dimensions of a basin for a given basin storage volume can be accomplished by rearranging equation 10.7 as shown in equation 10.8. The use of equation 10.8 is demonstrated in example 10-2.

$$L = \frac{-ZD(r+1) + [(ZD)^2(r+1)^2 - 5.33(ZD)^2 r + \frac{4rV}{D}]^{0.5}}{2r} \qquad (10.8)$$
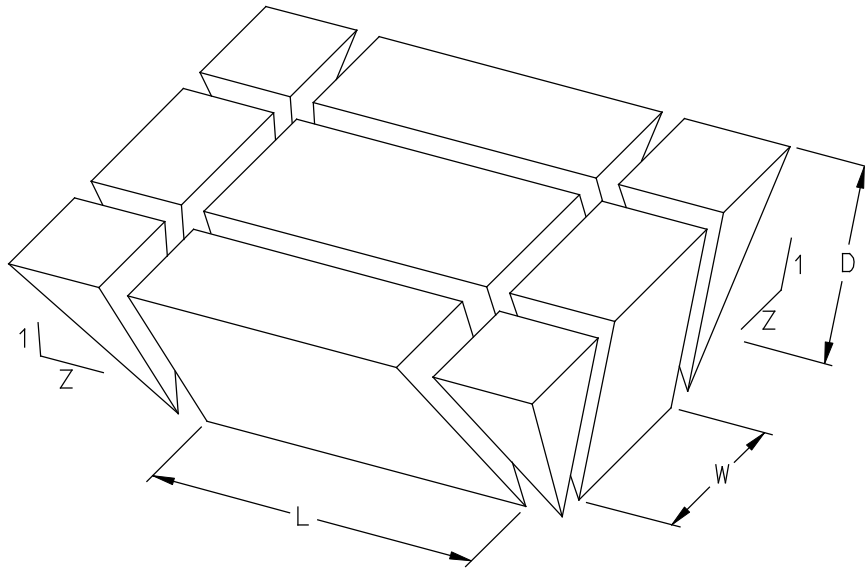

Figure 10-7 Trapezoidal Basin

10.7.4   Pipes and Conduits

If pipes or other storm drain conduits are used for storage, positive slope should be provided to transport sediment. This complicates storage calculations. The prismoidal formula presented in equation 10.9 can be used to determine the volume in sloping storm drain pipes. Figure 10-8 provides a definition sketch for the terms in equation 10.9.

$$V = \frac{L}{6}(A_1 + 4M + A_2) \qquad \textbf{(10.9)}$$

where: V   =   volume of storage, m^3 (ft^3)
         L   =   length of section, m (ft)
         A_1 =   cross-sectional area of flow at base, m^2 (ft^2)
         A_2 =   cross-sectional area of flow at top, m^2 (ft^2)
         M   =   cross-sectional area of flow at midsection, m^2 (ft^2)
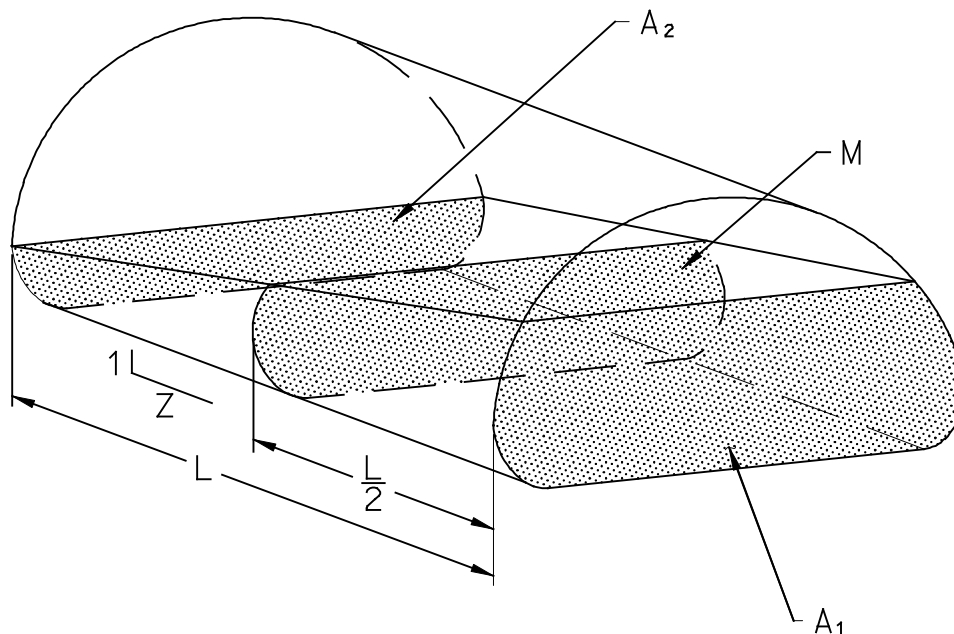
Figure 10-8 Definition Sketch for Prismoidal Formula

Calculations will be simplified if circular pipes are used, since the ungula of a cone and cylinder formulas may be used. The storage volume in a circular pipe, illustrated in Figure 10-9, can be computed using equation 10.10.

$$V = H \frac{(2/3\,a^3 \pm c\,B)}{r \pm c}$$ **(10.10)**

where: V =   volume of ungula, m^3 (ft^3)
         B =   cross-sectional area of flow, m^2 (ft^2)
         H =   wetted pipe length, m (ft)
         r =   pipe radius, m (ft)
                a, c, and a (radians) are as defined in Figure 10-9 (a and c have units of m (ft)) and

$$a = \sqrt{(2\,r\text{-}d)\,d}$$ **(10.11)**

$$c = d\text{-}r$$ **(10.12)**

$$\boldsymbol{a} = 2\,\sin^{-1}\left(\frac{a}{r}\right)$$ **(10.13)**

where: d =   flow depth in pipe, m (ft)

To assist in the determination of the cross-sectional area of the flow (B), equation 10.14 can be used to find the area of the associated circular segment.

$$A_s = (a - \sin a)\frac{r^2}{2}$$ **(10.14)**

where: $A_s =$ the segment area, m^2 (ft^2)

Using equation 10.10, the wetted area is computed as follows:

For $d \leq r$; $B = A_s$
For $d > r$; $B = A - A_s$

Alternatively, various texts contain tables and charts which can be used to determine the depths and areas described in the above equations.
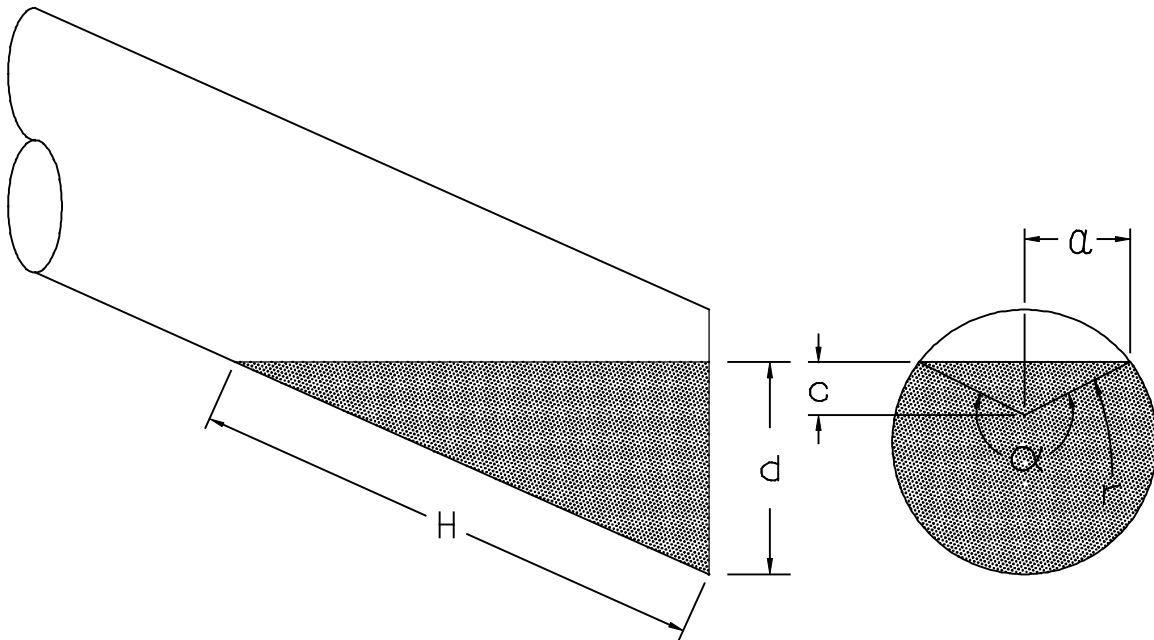

Figure 10-9 Ungula of a Cone

10.7.5   Natural Basins:

The storage volume for natural basins in irregular terrain is usually developed using a topographic map and the double-end area or frustum of a pyramid formulas. The double-end area formula is expressed as:

$$V_{1,2} = [\,(\,A_1 + A_2\,)\,/\,2\,]\,d \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textbf{(10.15)}$$

where: $V_{1,2}$ = storage volume between elevations 1 and 2, m^3 (ft^3)
$\quad\quad\quad$ A_1 = surface area at elevation 1, m^2 (ft^2)
$\quad\quad\quad$ A_2 = surface area at elevation 2, m^2 (ft^2)
$\quad\quad\quad$ d = change in elevation between points 1 and 2, m (ft)

The frustum of a pyramid is shown in Figure 10-10 and is expressed as:

$$V = d\,[\,A_1 + (\,A_1\,A_2\,)^{0.5} + A_2\,]\,/\,3 \qquad\qquad\qquad\qquad\qquad\qquad \textbf{(10.16)}$$

where: V = volume of frustum of a pyramid, m^3 (ft^3)
$\quad\quad\quad$ A_1 = surface area at elevation 1, m^2 (ft^2)
$\quad\quad\quad$ A_2 = surface area at elevation 2, m^2 (ft^2)
$\quad\quad\quad$ d = change in elevation between points 1 and 2, m (ft)
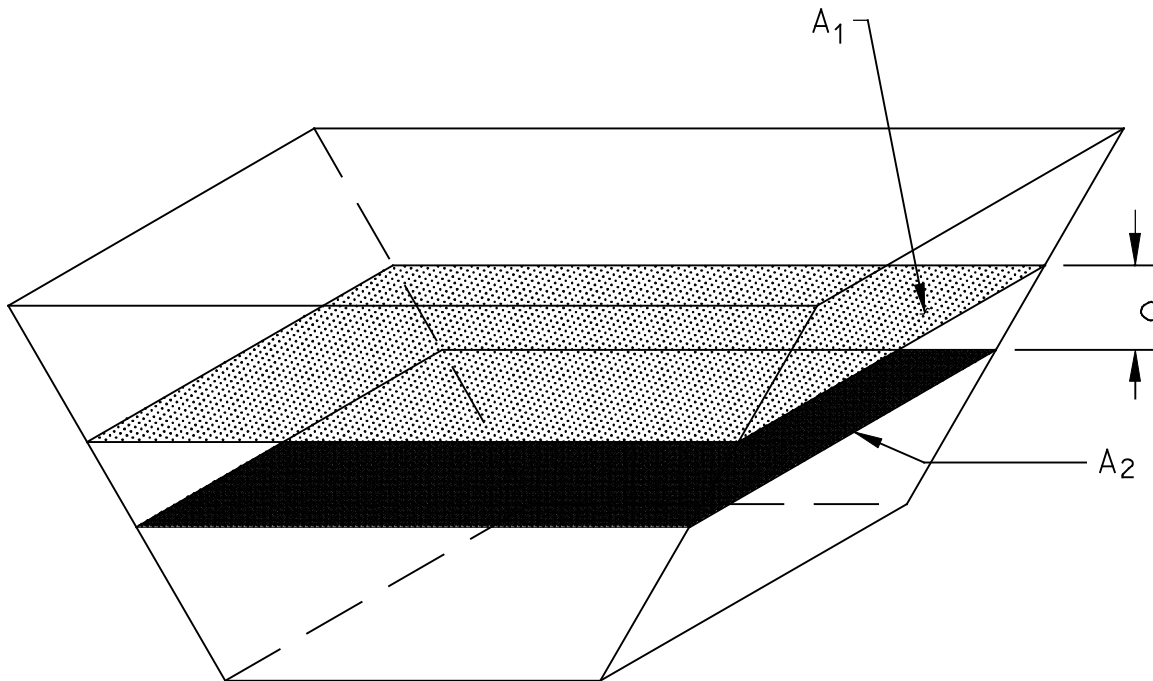

Figure 10-10 Frustum of a Pyramid

The following examples illustrate the development of a stage-storage relationship.

Example 10-2

Given: *Estimated Storage Volume (V) = 849 m³ (29,980 ft³) (from the triangular hydrograph from example 10-1 since it is the middle value). Use V = 850 m³ and 30,017 ft³.*
Depth Available for Storage During 10-yr Event (D) = 1.6 m (5.3 ft)
Available Freeboard = 0.6 m
Basin Side Slopes (Z) = 3 (V:H = 1:3)
Width to Length Ratio of Basin (r) = 1/2

Find: *(1)The length (L) and width (W) of basin at its base.*

(2)Develop a stage-storage curve for the basin assuming that the base elevation of the basin is 10.0 m (32.8 ft) and the crest of the embankment is at 12.2 m (40.0 ft). (This crest elevation is determined by adding the 1.6 m of available depth plus the 0.6 m of freeboard.)

Solution: *(1)   Substituting the given values in equation 10.8 yields the following:*

$$L = \frac{-ZD(r+1) + [(ZD)^2(r+1)^2 - 5.33(ZD)^2 r + \frac{4rV}{D}]^{0.5}}{2r}$$

$$L = \frac{(-3)(1.6)(0.5+1) + [(3\bullet 1.6)^2(0.5+1)^2 - 5.33(3\bullet 1.6)^2(0.5) + \frac{4(0.5)(850)}{1.6}]^{0.5}}{2(0.5)}$$

L = 25.2 m (82.7 ft)
Use L = 26 m (85 ft)
W = 0.5 L = 13 m (42.7 ft)

Use 26 m by 13 m basin (85 ft by 43 ft)

(2)By varying the depth (D) in equation 10.7, a stage-storage relationship can be developed for the trapezoidal basin. The following table summarizes the results:

DEPTH		STAGE		STORAGE VOLUME	
(m)	*(ft)*	*(m)*	*(ft)*	*(m³)*	*(ac-ft)*
0.00	0.0	10.0	32.8	0	0.000
0.20	0.7	10.2	33.5	72	0.059
0.40	1.3	10.4	34.1	155	0.125
0.60	2.0	10.6	34.8	248	0.201
0.80	2.6	10.8	35.4	353	0.286
1.00	3.3	11.0	36.1	470	0.381
1.20	3.9	11.2	36.7	600	0.486
1.40	4.6	11.4	37.4	744	0.603
1.60	5.2	11.6	38.0	902	0.731
1.80	5.9	11.8	38.7	1075	0.871
2.00	6.6	12.0	39.4	1264	1.024

Figure 10-5 illustrates the stage-storage relationship.

Example 10-3

Given: Given a storm drain pipe having the following properties:

>   Diameter = 1500 mm (60 in)   Pipe Slope = 0.01 m/m (ft/ft)
>   Pipe Length = 250 m (820 ft)   Invert Elevation = 30 m (98 ft)

Find: Develop a stage-storage tabulation between elevations 30 m (98 ft) and 31.5 m (103 ft)

Solution: Solve for the volume of storage using equations 10.10 and 10.14.

$$V = H \frac{(2/3\,a^3 \pm c\,B)}{r \pm c}\,, \qquad A_s = (a - \sin a)\frac{r^2}{2}$$

Note that: $B = A_s$ for $d \pounds r$
             $B = A - A_s$ for $d > r$
where A_s is the segment area and A is the total pipe area. The solution is provided in tabular form as follows:

Stage –Storage Tabulation

d		a		c		H		alpha	B		V	
(m)	(ft)	(m)	(ft)	(m)	(ft)	(m)	(ft)	(rad)	(m^2)	(ft^2)	(m^3)	(ft^3)
0.00	0.0	0.00	0.00	-0.75	-2.46	0.0	0.0	0.000	0.00	0.0	0.00	0.0
0.20	0.7	0.51	1.67	-0.55	-1.80	20.0	66.0	1.495	0.14	1.5	1.18	41.6
0.40	1.3	0.66	2.18	-0.35	-1.15	40.0	131.0	2.171	0.38	4.1	6.31	222.8
0.60	2.0	0.74	2.41	-0.15	-0.49	60.0	197.0	2.739	0.66	7.1	16.69	588.8
0.80	2.6	0.75	2.45	0.05	0.16	80.0	262.0	3.008	0.96	10.3	32.87	1159.9
1.00	3.3	0.71	2.32	0.25	0.82	100.0	328.0	2.462	1.25	13.5	54.98	1940.0
1.20	3.9	0.60	1.97	0.45	1.48	120.0	394.0	1.855	1.52	16.3	82.67	2917.3
1.40	4.6	0.37	1.23	0.65	2.13	140.0	459.0	1.045	1.72	18.5	115.09	4061.1
1.50	4.9	0.00	0.00	0.75	2.46	150.0	492.0	0.000	1.77	19.01	132.54	4676.9